



- Whole Numbers
- Representation of whole numbers on number line.
- Consecutive whole numbers
- Properties and Operations on whole numbers.

2.1. Introduction

You have already learned about natural numbers. Can we use these numbers only? This is a topic of discussion. As per requirement the group of natural numbers has to be expanded into group of whole numbers and by viewing carefully the various operations (addition, subtraction, multiplication and division) on the whole number we can see their rules, by applying these rules at proper places operations can be made easy.

2.2. Whole Numbers

Come let us see this situation

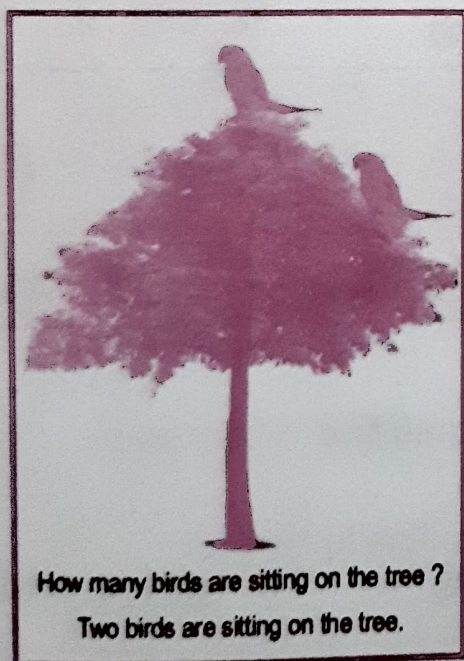


Fig. 1



Fig. 2



Fig. 3

You can tell the number of birds sitting on the tree in Fig. 1. and Fig. 2, but can you tell the number of birds on the tree in Fig. 3 by any number? the numbers known by us so far (1, 2, 3 ...) has no such number through which we can tell the bird sitting on the tree in Fig. 3.

We express it by number 0 (zero). That means we need number 0. Let us know about the numbers 0, 1, 2, 3,

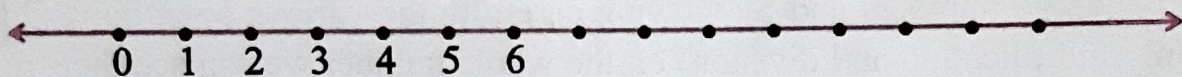
Let us take 10 balls in a box. Distribute these balls among 10 students equally. Each student got 1-1 ball. How many balls are left in the box ? Clearly, there is no ball in the box. Number of balls remaining in the box will be zero '0'. When we take zero with natural numbers then the number obtained is called the **Whole numbers**.

In his way

0, 1, 2, 3, 4 are whole numbers. '0' (zero) is the smallest whole number. Since every whole number there is whole number which is larger than that number. Therefore no whole number is the largest whole number.

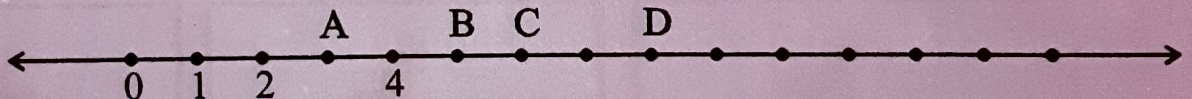
2.3 Representation of Whole Numbers on the Number Line.

Represent whole numbers starting with 0 with equal distance on the number line. In this way



Practice

1. See the number line below and tell which whole number is denoted by points A, B, C and D.



2. Tell the predecessor whole number of 1.
3. Tell the predecessor whole number of 13.
4. What is the successor whole number of 14 ?

2.4 Consecutive Whole Numbers

Focus on each group of the following numbers :

2, 3, 4 ; 7, 8, 9 ; 15, 17, 20

The number of first and second group are increasing from left to right with the difference of 1-1 respectively. How ?

Let's take a group of numbers more than three numbers 8, 9, 10, 11, 12. What is the feature of the numbers of this group ?

Numbers of the this group are also increasing with the difference of 1-1 respectively. Any following number in the group is the subsequent number of the previous number. The

numbers in any group with this characteristic is known as consecutive numbers. In this way numbers of first group and second group 2, 3, 4 and 7, 8, 9 are also consecutive numbers.

Are the above mentioned numbers of the third group 15, 17, 20 are consecutive numbers ? Clearly not, because 15, 17, 20 respectively are not increasing with the difference of 1-1. In other words 17 is not the following number of 15 and 20 is not the following number of 17.

Remark

In this way 13, 14 and 15 are the whole numbers in one sequence. These numbers are called consecutive whole numbers. Difference of two consecutive whole numbers is always one.

Practice

1. Tell two consecutive whole numbers just before 100.
2. Tell the four consecutive whole numbers starting with 101.
3. Take three consecutive numbers 3, 4 and 5. Add the first and third number, half the sum. Compare the middle number with the answer.

You will find that :

In three consecutive whole numbers, half of the sum of first and third numbers is the middle number.

Example 1. The sum of first and third number in three consecutive whole numbers is 40. Find the numbers.?

Answer : Sum of first and third number = 40

$$\therefore \text{Half of sum of first and third number} = \frac{40}{2} = 20$$

$$\therefore \text{Middle number} = 20$$

So, numbers are 19, 20, 21

Exercise 2 (a)

1. Which is the smallest whole number ?
2. Represent the following whole numbers of the number line 0, 1, 2, 3, 4, 5
3. In three consecutive whole number, sum of first and third number is 28. Find the numbers.
4. If in three consecutive whole numbers, middle number is 39, then find three numbers.
5. Which number is included in the set of natural numbers so that it becomes whole number ?

2.5 Properties and Operations on Whole Numbers

2.5.1 Addition properties of Whole Numbers

(i) Closure property of addition

Take pair of whole numbers for example—

(5, 7), (3, 4), (0, 3) and (0, 0)

Find the sum of numbers of each pair.

Whole Number + Whole Number	Sum	Is it a whole number or not
$5 + 7$	12	A whole number
$3 + 4$		A whole number
$0 + 3$		A whole number
$0 + 0$		A whole number

Test, similar three other pairs of whole numbers and find their sum. Is the sum always a whole number ?

Did you find a pair of whole numbers whose sum is not a whole number ?

Any two whole number can not be found whose sum is not a whole number.

Conclusion

Sum of two whole numbers is always a whole number. This is closure property in addition of whole number.

(ii) Commutative property of addition :

Radha first buys 3 toffees from a shop and again 2 toffees. How many toffees did Radha buy all together ?

Mahmood also buys toffees from the same shop. He first buys 2 toffees and then 3 toffees. How many toffees did Mahmood buy ?

Radha and Mahmood bought 5 toffees each.

Take any two whole numbers by adding them fill in the blanks

$$3 + 5 = \square$$

Again change the order of the number and then find their sum.

$$5 + 3 = \square$$

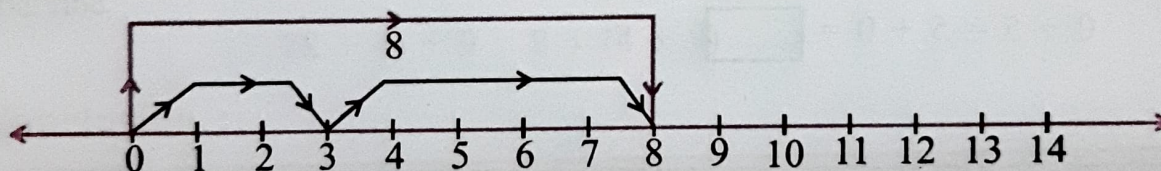
Compare the sum of both the situations

$$3 + 5 = 8 = 5 + 3$$

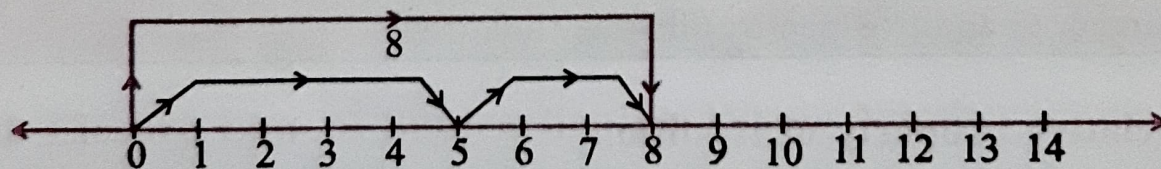
We see that

$$3 + 5 = 5 + 3$$

This fact, we can understand on the number line also. Draw number line and find the sum of $3 + 5$



Similarly draw another number line and find the sum of $5 + 3$



We find that in both the situation the sum on number line are the same.

$$\text{Therefore } 3 + 5 = 5 + 3$$

Test :

Take two other whole numbers like (3, 4) and (6, 2) check.



$$3 + 4 = 7 \text{ and } 4 + 3 = 7$$

Therefore, $3 + 4 = 4 + 3$

Similarly $6 + 2 = 8 \text{ and } 2 + 6 = 8$

Therefore $6 + 2 = 2 + 6$

In this way we conclude that

The order of numbers do not have any effect on the sum of two whole numbers.

(iii) Additive identity

You have already seen that how is the collection of whole numbers different from the collection of natural numbers. It is just the presence of 'Zero' in the collection of whole numbers. This number 'Zero' has a special role in addition. Let us see to try for this.

Let us take some whole numbers such as 0, 1, 3, 5 and fill in the blanks.

$$0 + 0 = 0$$

$$0 + 1 = \boxed{}$$

$$0 + 3 = 3 + 0 = \boxed{}$$

$$0 + 5 = 5 + 0 = \boxed{}$$

Note it :

If zero is added in any whole number the sum will be the same number. This is why "zero" 0 is called an identity for addition of whole number or additive identity for whole numbers.

(iv) Associative property of Addition

If we want to find the sum of 2, 3 and 4 in the same order, then we can do it by two methods. Firstly find the sum of 2 and 3 and then add 4 to this or add 2 in the sum of 3 and 4. In both conditions we will find the same sum. For example—

$$(2 + 3) + 4 = 5 + 4 = 9$$

and $2 + (3 + 4) = 2 + 7 = 9$

Let's check this on number line.

With the help of number line, show the sum of 2, 3 and 4 in two ways.

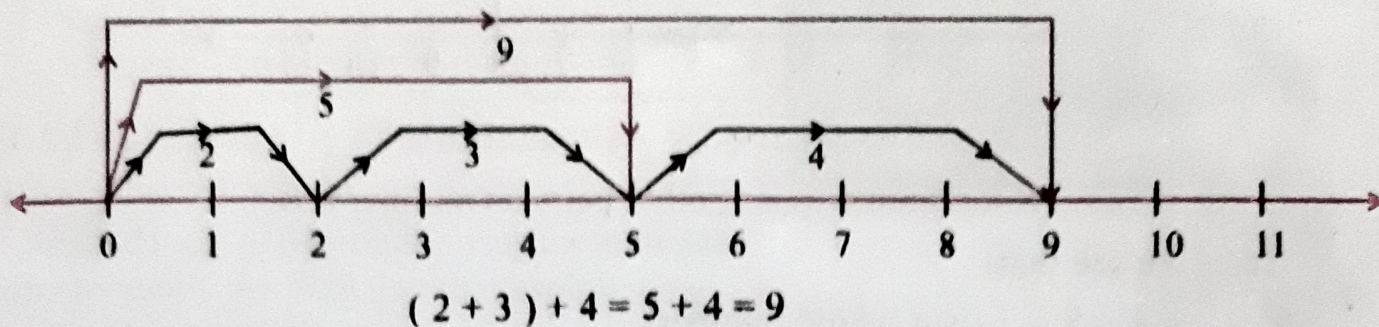


Fig. 1

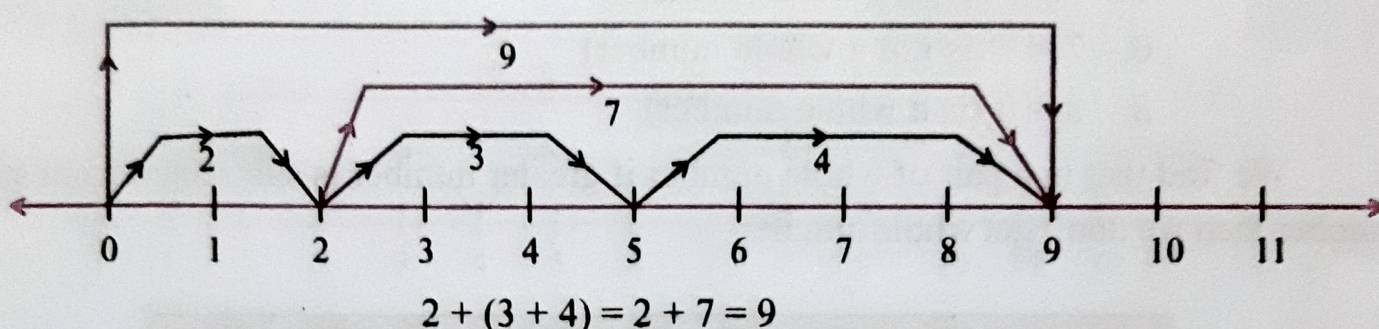


Fig. 2

In fig. 1 and fig. 2, we find that

$$(2 + 3) + 4 = 2 + (3 + 4)$$

in the same way, when we add whole numbers of order 8, 4, 9 we will find on the number line.

$$(8 + 4) + 9 = 8 + (4 + 9)$$

✎ Conclusion :

During the addition of three whole numbers in order making group of any two whole numbers does not make any difference in the sum. This is the associative property of addition.

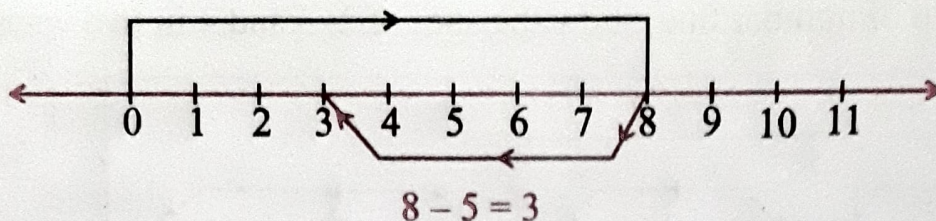
2.5.2 Closure Law of Subtraction on Whole Number

Take some pair of whole numbers like

8, 5; 5, 9; 10, 0; 0; 7; and 8, 8

Subtract the second number from the first in every pair.

Represent 8-5 on the number line in below mentioned way :



In this way show the subtraction of other pairs on number line.

Here, we see that-

$8 - 5 = 3$ (is a whole number)

$5 - 9 = ?$ (is not a whole number)

$10 - 0 = 10$ (is a whole number)

$0 - 7 = ?$ (is not a whole number)

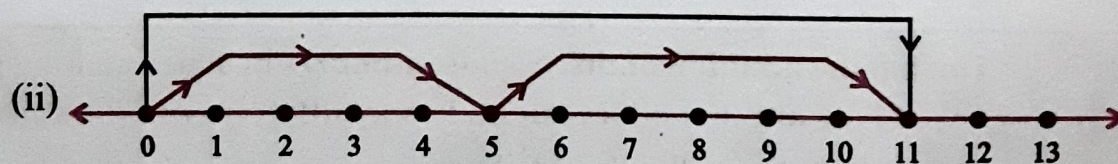
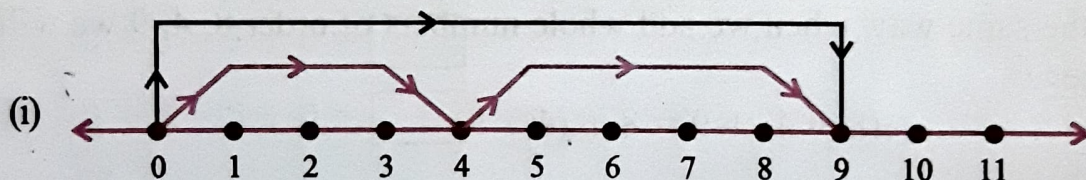
$8 - 8 = 0$ (is a whole number)

We find that in a pair of whole number if greater number is subtracted from smaller number then we don't get whole number.

Thus whole numbers are not closed under subtraction.

Exercise 2 (b)

1. Write the addition fact described on the number line.



2. Show the following facts on numbers line.

(i) $5 + 4 = 9$,

(ii) $0 + 5 = 5$,

(iii) $2 + 3 + 4 = 9$

3. Fill in the blanks in the following statements :

(i) $345 + 789 = \boxed{} + 345$

(ii) $2889 + 0 = \boxed{}$

(iii) $(234 + 456) + 789 = \boxed{} + (456 + 789)$

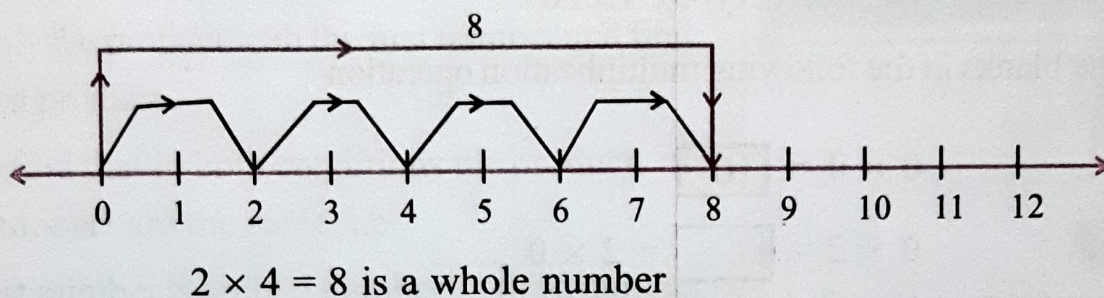
4. Subtraction is not commutative for whole numbers. Use three different pairs of numbers and verify it.

2.5.3. Properties of Multiplication in Whole Numbers :

(i) Closure property of Multiplication

Take pairs of whole numbers e.g. (0, 8), (5, 3), (7, 13) and (6, 5) and by multiplying the numbers of the pairs mutually, we find that the result in every condition is a whole number.

Therefore whole numbers follow the closure property of multiplication. This can be understand as follows.



$0 \times 8 = 0$ (a whole number)
 $5 \times 3 = 15$ (a whole number)
 $7 \times 13 = 91$ (a whole number)
 $6 \times 5 = 30$ (a whole number)



Try it

Multiply the pairs of whole number e.g. (for example) 6, 4 ; 5, 8; and 3, 10 each on number line and find is the result a whole number?

Conclusion :

Multiplication of whole numbers are always whole number. This is closure property of multiplication operation.

(ii) Commutative property of Multiplication

Take a few pairs of whole numbers (3, 5), (2, 7) and (9, 13), compare the results obtained by multiplying the first number to the second number and second number to the first number. Even, after changing the order of pair of whole number the product is the same. i.e.,

$$3 \times 5 = 15$$

$$5 \times 3 = 15$$

Conclusion :

Even, after changing the order of pair of whole numbers, the product remains the same. This is the commutative property of multiplication operation.

Try it

Prove the commutative property of multiplication by the pair 2, 3; 12, 5 ; 6, 4 and 0, 7.

(iii) Multiplication Property of Zero

Fill the blanks in the following multiplication operation

$$0 \times 0 = \boxed{0}$$

$$0 \times 2 = \boxed{} = 2 \times 0$$

$$3 \times 0 = \boxed{} = 0 \times 3$$

Try it

Take few whole numbers and observe the product obtained by multiplying them with zero. Is the product every time zero ?

Conclusion :

Product of any whole number and zero is always zero.

(iv) Multiplicative Identity

Fill the blanks in the following multiplication number table.

$1 \times 0 = 0 \times 1 =$	
$1 \times 1 =$	
$1 \times 2 = 2 \times 1 =$	
$1 \times 3 = 3 \times 1 =$	

Practice

Take few more whole numbers i.e, 5, 8, 17, 31 etc. and multiply them with 1. Each time you will find that the product is the same number by which 1 is multiplied.

✌ Conclusion :

Product of any whole number and 1 is the number itself.

1 is called multiplicative identity.

(v) Associative Property of Multiplication :

Take any three whole numbers Ex. 3, 8 and 5 first of all multiply first number with second after finding the product again multiply by the third. Write the resultant product of all three multiply the second and third number then multiply the product with the first number and find the resultant product.

We find that in both conditions the products of all three numbers are the same. i.e.

$$\begin{aligned}(\text{first number} \times \text{second number}) \times \text{third number} \\ = \text{first number} \times (\text{second number} \times \text{third number})\end{aligned}$$

In the same way check by taking other three whole numbers.

✌ Conclusion :

In continuous multiplication operation of any three whole numbers the product is not changed by the changing the order. Therefore, multiplication operation is associative in whole numbers.

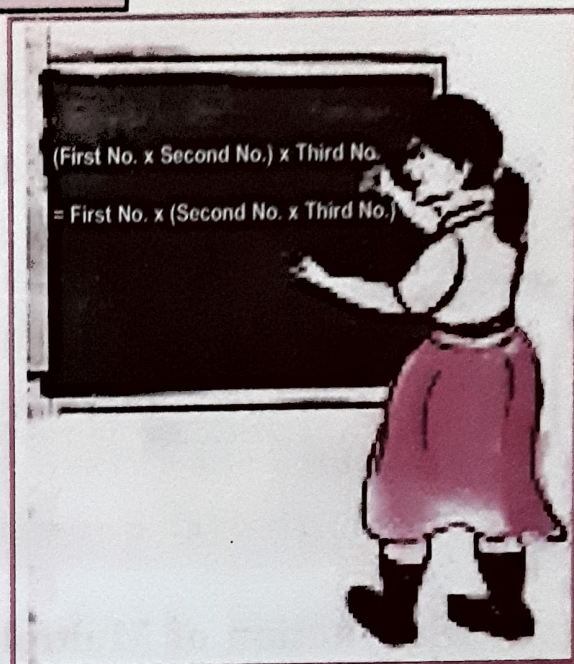
✂ Try it

Take whole numbers 6, 9 and 11. Verify associative property of multiplication.

(vi) Distribution of Multiplication Operation over Addition :

Take whole number 3, 6 and 8 and show the multiplication of first number with the sum of second and third is the same as the sum of product of first number with second number and product of first number with third number.

Similarly, take three other whole numbers and verify.



Conclusion :

For any three whole numbers—

$$\text{First Number} \times (\text{Second Number} + \text{Third Number})$$

$$= \text{First Number} \times \text{Second Number} + \text{First Number} \times \text{Third Number}$$

This is called property of distribution of multiplication over addition.

Try it :

Test the truth of following statements-

(i) $9 \times (8 + 4) = (9 \times 8) + (9 \times 4)$

(ii) $6 \times (10 + 7) = (6 \times 10) + (6 \times 7)$

(iii) $11 \times (12 + 8) = (11 \times 12) + (11 \times 8)$

(vii) Distribution of Multiplication over Subtraction

Let us take whole numbers 4, 8 and 5 and show that

$$\text{First Number} \times (\text{Second Number} - \text{Third Number}) = \text{First number} \times$$

$$\text{Second number} - \text{First number} \times \text{Third number}$$

Therefore, distribution of multiplication over subtraction is applicable.

Exercise 2(c)

1. Fill in the blanks in your note book according to the properties of the multiplying operation in the following statements.
 - (i) $468 \times 0 = \boxed{}$
 - (ii) $8976 \times 5432 = 5432 \times \boxed{}$
 - (iii) $8973 \times 1 = \boxed{}$
2. Find the product at the following using suitable properties :
 - (i) $4 \times 2834 \times 25$
 - (ii) $5 \times 658 \times 80$
 - (iii) $25 \times 4837 \times 40$
3. Find the product using distributive properties in each of the following :
 - (i) 487×1008
 - (ii) 998×436
 - (iii) 6754×94
 - (iv) 26478×106

2.5.4 Divisional Operation on Whole Numbers :

Division operation is opposite of multiplication operation. For example, $4 \times 5 = 20$, then $20 \div 4 = 5$ and $20 \div 5 = 4$. Now consider the properties of division operation.

(i) Experiment of closure property in division operation

Take pairs of whole numbers

Example—24, 6; 17, 4; 7, 8; 9, 5; & 9, 9

In every pair divide the first number by the second.

We find that in every situation dividing the first whole number by the second whole number. We do not get the whole number.

The division operation is not closed for whole number.

(ii) Test of division of whole numbers by zero

By method of subtraction, let us try $8 \div 0$.

$$\begin{array}{r} 8 \\ -0 \\ \hline 8 \end{array} \quad \text{subtracting for the first time}$$
$$\begin{array}{r} -0 \\ \hline 8 \end{array} \quad \text{subtracting two times}$$
$$\begin{array}{r} -0 \\ \hline 8 \end{array} \quad \text{subtracting three times}$$

In this way, we seen that we never get 0 at any stage of subtraction.
Therefore,

Division of a whole number by 0 is not defined.

(iii) Test of division of whole numbers by 1.

See the operations of division of the following

$$2 \div 1 = 2$$

$$3 \div 1 = 3$$

$$4 \div 1 = 4$$

Dividing any whole number by 1 the quotient remains the same number.

(iv) Test of division of zero by any non-zero number.

We see that

$$0 \div 5 = 0$$

$$0 \div 6 = 0$$

The quotient will be always zero when you divide zero by a non zero whole number.

(v) Test of division of any non-zero number by the same non-zero number.

We find that :

$$2 \div 2 = 1$$

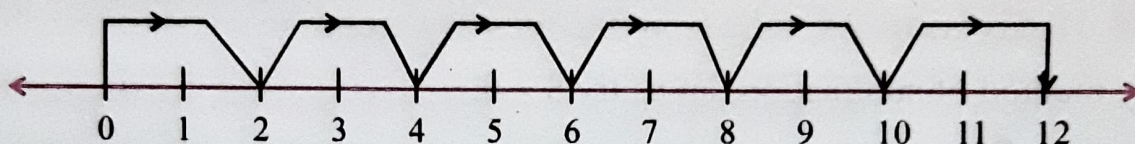
$$3 \div 3 = 1$$

$$4 \div 4 = 1$$

When a non zero whole number is divide by the same whole number, the quotient is always 1.

Exercise 2 (d)

1. Tell the facts divided by the following number line.



2. The multiplicative fact of $21 \div 3 = 7$ is $3 \times 7 = 21$. Therefore find the multiplicative fact of the following facts—

(i) $56 \div 8 = 7$

(ii) $66 \div 11 = 6$

3. Express 117 as product of two numbers in which one number is 13.
4. Is such a whole number possible, in which the same number is obtained by dividing it by itself ?
5. Will the quotient be the same when two different non-zero whole numbers are divided first number by second number and second number by first number ?

National Income and eligibility based scholarship question :

1. Non divisible number between 80 and 90 is—
(a) 81 and 83 (b) 83 and 87 (year 2005)
(c) 81 and 89 (d) 83 and 89 Ans. (d)
2. There are five terms in this question. Four terms are similar in some form and one term is different from the rest. Circle the different term against the relevant question in your note-book.
(a) 6, 3, 18 (b) 7, 5, 35
(c) 9, 3, 27 (d) 5, 4, 26
(e) 8, 7, 56 Ans. (d)

3. Find out the missing terms from the given five options and write the answer in front of the right question on the answer sheet—

196, 169, 144, 121, 100 ?

(a) 85

(b) 90

(c) 81

(d) 64

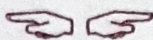
(e) 95

Ans. (c)

We Learnt in this Unit

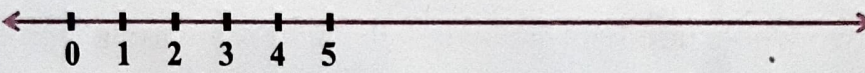
1. In sequential orders an incoming number is called consecutive numbers.
2. By including zero to natural numbers, the number obtained are whole numbers.
3. Zero is the smallest whole number.
4. None of the whole number is the biggest whole number as it also has a its successor whole number.
5. When two whole numbers are added, the sum is always a whole number. This is the **closure property** of whole numbers.
6. The sequence of numbers does not have any effect on the sum of two whole numbers. This is the **commutative property** of addition.
7. When adding 0 is added to a whole number the sum is the number itself. Therefore 0 is called **additive identity element** of the whole number.
8. When adding three whole numbers, grouping any two numbers first does not make any difference to sum. This is the **associative property** of addition operations.
9. Whole numbers are not closed under subtraction.
10. The product of two whole numbers is always the whole number. This is the **closure property** of multiplication in whole numbers.
11. There is no effect in ordering to multiply two whole numbers. This is the **commutative property** of multiplication operation in whole numbers.
12. Product of any whole number and zero is always zero. This is the multiplication property of zero.
13. Product of any whole number by 1 is always the same number. Hence 1 is the **identity element of multiplication** in whole number.
14. There is no change of product due to the continuation of any continuous multiplication of operation of any three whole numbers. This is the **associative property** of multiplication in whole numbers.

15. In whole numbers, the multiplication operation is distributed on both the sum and or difference operation.
16. The operation of division is not closed for the whole number.
17. Division is not defined by "0" on whole number.
18. In dividing any whole number by 1, the quotient is always the same number itself.
19. On dividing zero by any non-zero number, the quotient always comes zero.
20. By dividing any non-zero whole number by the same non-zero whole number the quotient always comes 1.



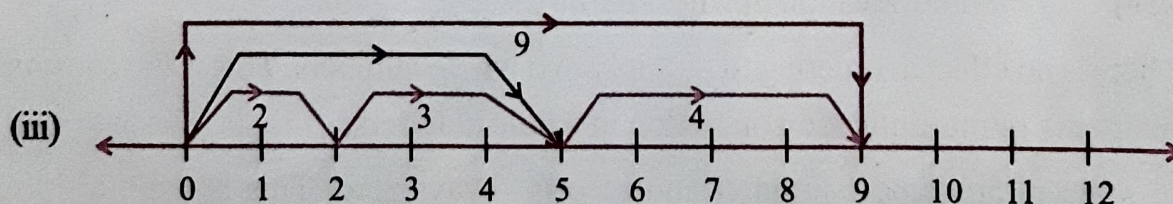
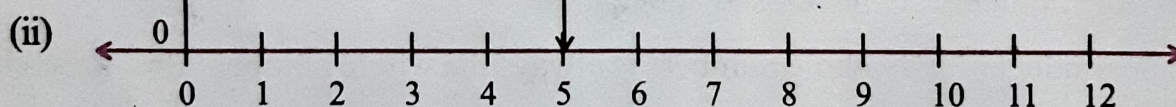
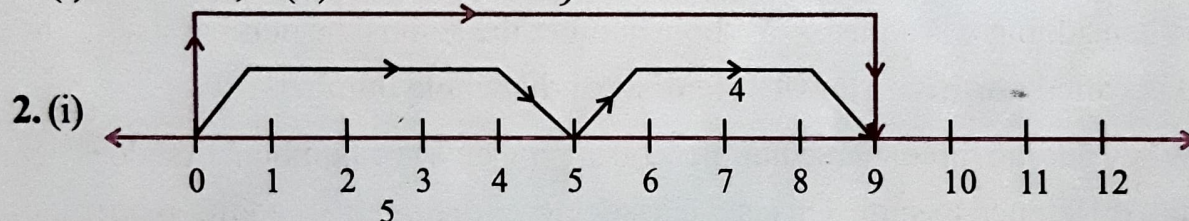
ANSWERS

Exercise—2 (a)

1. 0; 2. 
 3. 13, 14, 15, 4. 38, 39, 40 5. 0

Exercise—2 (b)

1. (i) $4 + 5 = 9$, (ii) $5 + 6 = 11$



3. (i) 789, (ii) 2889, (iii) 234

Exercise—2 (b)

1. (i) 0, (ii) 8976, (iii) 8973 2. (i) 283400, (ii) 263200, (iii) 4837000;
 3. (i) 490896, (ii) 435128 (iii) 634876 (iv) 2806668.

Exercise—2 (d)

1. $12 \div 2 = 6$; 2. (i) $8 \times 7 = 56$, (ii) $11 \times 6 = 66$; 3. $117 = 13 \times 9$; 4. Yes, This number is one, 5. No.

