

UNIT-10 LEAST COMMON MULTIPLE AND HIGHEST COMMON FACTOR



- Factor
- Multiple
- Factor, Common factor and Highest Common factor (HCF)
- Multiple, Common multiple, least common multiple (LCM)
- To find HCF by division method
- The relation between two numbers and its HCF and LCM.
- Use of HCF and LCM in practical life.

Introduction

You know about natural numbers, whole numbers and set of integers very well. You also know about the operation of addition, subtraction, multiplication and division of these numbers. It is clear that every number can be written as product of two or more than two numbers by the methods of multiplication. In the same way it is also clear that every number is divisible by one or itself too or there is some remainder after the division by the method of division. Now we try to see that a natural number can be written as product of which numbers and to find the complete divisors of natural number? The numbers which divide a natural number and that number is a multiple of each of its divisors. For example : The complete divisors of 15 are 1, 3, 5 and 15 which are divisors of 15 and 15 is a multiple of 1, 3, 5 and 15. In this unit we will study about the divisors, common divisors and greatest common divisors of natural numbers and also we will study about multiples, common multiples, least common multiple of natural numbers. We will also study about product of two numbers, the relation between the highest common factors and the least common multiple.

10.2 Factors or Divisors

Shila has 12 marbles. She arranges these in rows such that each row has equal numbers of marbles. She arranges these as the following methods and counts the total number of marbles as :

(1) One marble in each row; number of rows = 12

$$\text{Total number of marbles} = 1 \times 12 = 12$$

(2) Two Marbles in each rows, number of rows = 6



Total number of marbles = $2 \times 6 = 12$

- (3) 3 marbles in each row :

Total number of rows = 4

Total number of marbles = $3 \times 4 = 12$

- (4) 4 marbles in each row

Total number of rows = 3

Total number of marbles = $4 \times 3 = 12$

- (5) 5 marbles in each row then it is not possible to put 5 marbles in each row which is shown the adjacent figure.

- (6) 6 marbles in each row number of row = 2

Total number of marbles = $6 \times 2 = 12$

- (7) 7, 8, 9, 10 and 11 marbles respectively in each row. There is no way to arrange equal number of marbles in each row in these cases, but 12 marbles can be arranged in a row

Number of row = 1

Total number of marbles = $1 \times 12 = 12$

By these calculation, Shila sees that 12 can be written as product of two numbers in the different ways 1.

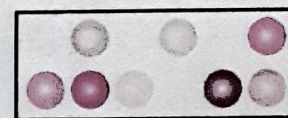
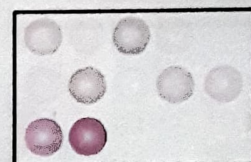
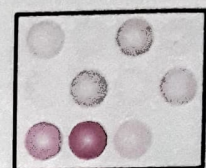
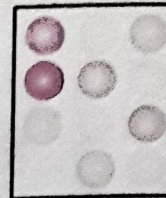
$12 = 1 \times 12$; $12 = 2 \times 6$; $12 = 3 \times 4$; $12 = 4 \times 3$;

$12 = 6 \times 2$; $12 = 12 \times 1$

Thus, 1, 2, 3, 4, 6 and 12 are divisors of number 12.

These are called factors of 12.

When a given number is completely divisible by certain numbers, then those divisors are called factors of the given number.



Now we see divisors or some numbers in the given table, we think and find conclusion about some interesting facts of the factors by this methods.

Number	factors
2	1, 2
6	1, 2, 3, 6
15	1, 3, 5, 15
69	1, 3, 23, 69
84	1, 2, 3, 4, 6, 7, 12, 21, 28, 42, 84

Conclusion :

We see that :

- 1 is factor of each number
- Every number is factor of it self.
- Each factor of any number is a complete divisor of that number.
- The number of factor of a given number is finite.
- Every factor of a number is either less than or equal to that number.

10.3 Multiple

When we write $20 = 4 \times 5$, then we say that 4 and 5 are factors of 20. In another way we can also say that 20 is the multiple of 4 and 5.

$$4 \times 5 = 20 \text{ (multiple)}$$

(factor)

In similar way $24 = 2 \times 12$, it is seen that 2 and 12 are factors of 24 and 24 is a multiple of 2 and 12. We get different multiple of 2 and 12. We get different multiples of a number of multiplying the natural numbers (1, 2, 3) in that number.

e.g. multiples of 3 are 3, 6, 9,

multiples of 5 are 5, 10, 15, 20 etc.

Now, we can see some interesting facts of multiples form the following tables :

Number	Multiples
1	1, 2, 3,
7	7, 14, 21,
8	8, 16, 24, 32,
10	10, 20, 30,
12	12, 24, 36,

Conclusion :

We see that

- Any number is a multiple of its factors.
- Each multiple is greater than or equal to that number
- The number of multiples of a number is infinite.
- Each number is a multiple of itself.
- A number is multiply by natural numbers (1, 2, 3, ...) we get different multiples of that number.

Perfect number :

Now we discuss about the factors of some numbers :

e.g. All divisors of 6 are 1, 2, 3 and 6. Sum of these are $1 + 2 + 3 + 6 = 12$ which is two times of 6.

Similarly, All factors of 28 are 1, 2, 4, 7, 14 and 28.

Sum of these $1 + 2 + 4 + 7 + 14 + 28 = 56$, which is two times of 28.

Here, we see that sum of all factors of 6 and 28 are two times of 6 and 28 respectively.

A number, whose sum of all factors of that number is two times of that number is called a perfect number.

Therefore 6 and 28 are prefect number.

Find such type of some perfect numbers.

Exercise 10 (a)

- Match the divisors of column (2) and numbers of column (1)

Column 1

(1) 15

(2) 24

Column 2

(a) 1, 2, 4, 8, 16, 32

(b) 1, 2, 3, 4, 6, 9, 12, 18, 36

- (3) 21 (c) 1, 3, 5, 15
 (4) 32 (d) 1, 3, 7, 21
 (5) 36 (e) 1, 2, 3, 4, 6, 12, 24

2. Write first five multiples of following numbers :

- (a) 3 (b) 4 (c) 5 (d) 9

3. Match the column (2) to the numbers of column (1)

Column 1

Column 2

(1) 15

(a) multiple of 7

(2) 20

(b) multiple of 8

(3) 16

(c) divisor of 50

(4) 25

(d) divisor of 30

(5) 35

(e) divisor of 20

4. Find the all multiples of 7 less than 100.

5. Write all factors of 496 and show that it is a perfect number.

6. Fill in the blanks :

- (1) Each number isof
 (2) Complete divisor of a number is of that number.
 (3) Each factor of a number is or to that number.
 (4) Each number of its factor is a
 (5) Divisors of 78 are

10.4 Prime Numbers and Composite Numbers

Number	Divisors	Number of Divisors
1	1	1
2	1, 2	2
3	1, 3	2
4	1, 2, 4	3
5	1, 5	2
6	1, 2, 3, 6	4
7	1, 7	2
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
11	1, 11	2
12	1, 2, 3, 4, 6, 12	6

Some numbers as 2, 3, 5, 7, 11 etc have only two divisors (1 and itself) of that number. These numbers are called prime number.

Those numbers whose divisors are only 1 and itself that number, are called prime numbers.

Some numbers as 4, 6, 8, 9, 10 etc are those numbers which has two or more than two divisors, are called composite numbers. We see that :

Those numbers, which has two or more than two divisors are called composite numbers.

1 is a number whose divisor is only 1 so, 1 is not a composite number nor a prime number.

Conclusion :

Those numbers which are divisible by only 1 and itself are called prime numbers. Those numbers which are divisible by number other than one and itself are called composite numbers.

10.5 Prime Factor

We are aware about the method of factorisation of any number. Now we will take a few example to think about the prime factor.

$$24 = 2 \times 12$$

$$= 2 \times 2 \times 6$$

$$= 2 \times 2 \times 2 \times 3$$

$$24 = 4 \times 6$$

$$= 2 \times 2 \times 6$$

$$= 2 \times 2 \times 2 \times 3$$

$$24 = 3 \times 8$$

$$= 3 \times 2 \times 2 \times 2$$

$$= 2 \times 2 \times 2 \times 3$$

Now we will get only one factor 2 ' 2 ' 2 ' 3 at the last of three type of factorization of 24. In this factorization only 2 and 3 are the factors, these are prime numbers. Such a factor of a number is called prime factorization.

Let's do :

To see the prime factors of 84 :

$$84 = 2 \times 42$$

$$= 2 \times 2 \times 21$$

$$= 2 \times 2 \times 3 \times 7$$

$$84 = 6 \times 14$$

$$= 2 \times 3 \times 2 \times 7$$

$$= 2 \times 2 \times 3 \times 7$$

$$84 = 4 \times 21$$

$$= 2 \times 2 \times 21$$

$$= 2 \times 2 \times 3 \times 7$$

$$84 = 28 \times 3$$

$$= 2 \times 14 \times 3$$

$$= 2 \times 2 \times 7 \times 3$$

$$= 2 \times 2 \times 3 \times 7$$

$$84 = 12 \times 7$$

$$= 4 \times 3 \times 7$$

$$= 2 \times 2 \times 3 \times 7$$

At least we see that there are only four prime factors of 84 in each cases. It is of course that the other of the factor has changes some where.

This leads to the conclusion that :

The number of prime factors of a composite number is fixed, but the order of the factors may be different.

We have already learn to find the prime factors by division methods as :

2	84
2	42
3	21
	7

$$\therefore 84 = 2 \times 2 \times 3 \times 7$$

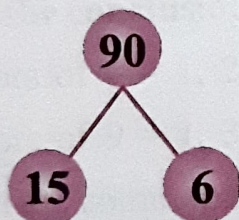
Write a number 90, Think about any pair of factors of 90

$$90 = 15 \times 6$$

Now think factors of 15 as :

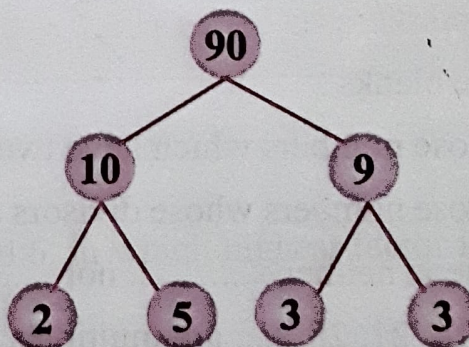
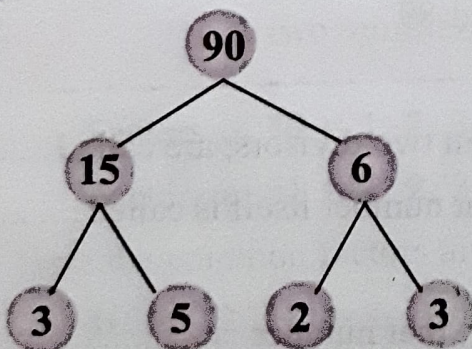
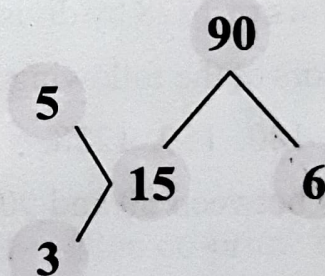
$$15 = 3 \times 5$$

write it of factors of 6



$\therefore$

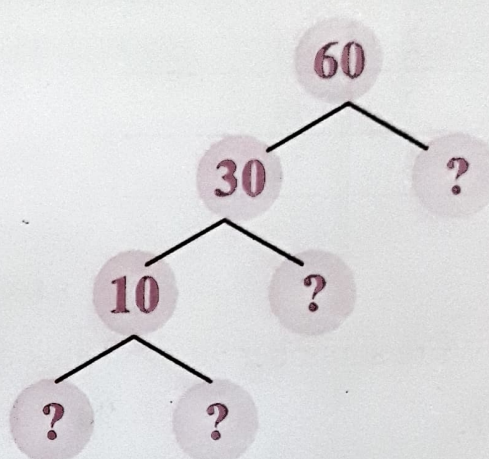
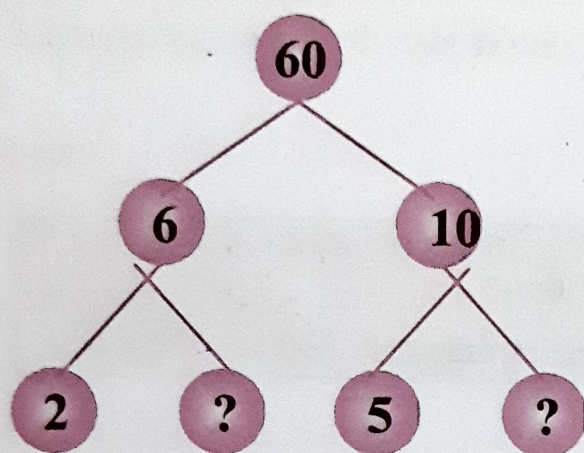
also



Therefore, $90 = 5 \times 3 \times 3 \times 2$ or, $90 = 2 \times 5 \times 3 \times 3$ or,
 $90 = 2 \times 3 \times 3 \times 5$ etc.

Exercise 10(b)

1. Write prime factors of 9.
2. If the prime factors of a number are 2, 2 and 3, then find the number.
3. If the factors of a number are 8 and 3. Find the prime factors of that.
4. Here two different factor trees of 60 are given. Write the unknown numbers on your notebook.



5. Write highest four digit number and represent it, as prime factors.
6. Write the smallest five digit number and represented it as prime factor.
7. Find prime factors of 1728 by division method.
8. Find prime factors of the following numbers :
112, 120, 135, 140, 150, 1228.
9. Prime numbers between 80 and 90 are :
(1) 81 and 83 (2) 81 and 89
(3) 83 and 87 (4) 83 and 89
10. Fill in the blanks :
(i) Those numbers which have two or more than two divisors, are called
(ii) Those numbers whose divisors are 1 and that number itself is called
(iii) One is neither nor
(iv) 7, 14, 21, 28 are multiples of a number, that number is

10.6 Factor, Common factor, highest common factor :

We know that a number which divides any number completely is called factor of that number. See the following chart and think.

Number	Factors
48	1, 2, 3, 4, 6, 8, 12, 16, 24, 48
64	1, 2, 4, 8, 16, 32, 64
72	1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

Here we see that 1, 2, 4 and 8 are the common factors of above given numbers, in which 8 is the greatest. Therefore Highest common factor of the numbers 48, 64 and 72 is 8. In short form, highest Common factor is written as HCF or greatest common divisor (G.C.D.).

The highest common factor is the highest, among all the common divisors of two or more than two given numbers.

Do it :

Write down all factors of 45, 60 and 75 and find its highest common factor.

Think and find the conclusion

Factors of 4 = 1, 2, 4

Factors of 9 = 1, 3, 9

Therefore the common factor is only 1, so

HCF of 4 and 9 is 1 such type of number are called co-prime numbers.

Those two numbers, whose HCF is 1, are called co-prime numbers.

Factors of 12 = 1, 2, 3, 4, 6, 12

Factors of 18 = 1, 2, 3, 6, 9, 18

Here the common factors are 1, 2, 3, and 6. in which highest factor is 6.

$\therefore \text{HCF}(12, 18) = 6$

Now, Factors of 8 = 1, 2, 4, 8

Factors of 16 = 1, 2, 4, 8, 16

Therefore, H. C. F of 8 and 16 = 8.

So, we get the conclusion from the above three examples.

H.C.F. of Co-prime numbers is always 1. Generally, H.C.F. of the numbers is the smaller than the smallest number among them or it is equal to the smallest number of these.

10.7 To Find Highest Common Factor by factor methods :

See the prime factors of 48, 64 and 72.

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3$$

The easiest method to find H.C.F. is :

- (1) First select the smallest prime factor from the prime factors of all the numbers. Write this factor number of times as it is common in the factor of the numbers.
- (2) In same way we write the bigger prime factor than that number. To write it as many times as it is common.
- (3) In same way write all the common factors.
- (4) H. C. F. is the multiplication of all these factors.

Therefore, H. C. F. of 48, 64 and 72 in the above example $= 2 \times 2 \times 2 = 8$

We can do this in the following way also,

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$$

$$64 = 2 \times 2 \times 2 \times 2 \times 2 = 2^6$$

$$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$

Here the minimum power of 2 is 3, which is common factor.

herefore $HCF = 2^3 = 2 \times 2 \times 2 = 8$

Example 1 : Find HCF of 180 and 192 by factor method.

Solution : We see that the following prime factors of the given numbers are :

$$180 = 2 \times 2 \times 3 \times 3 \times 5$$

$$192 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

We see that the common factors of factors of both numbers are 2 times 2 and one time 3.
So, required HCF = $2 \times 2 \times 3 = 12$

10.8 : To find HCF by division method.

It can be understood by the following examples :

Example 2 :

Find the HCF of 306 and 630 by division method.

Solution :

Here, bigger number 630 is divided
by smaller number 306.

Again, the first dividend is divided by the remainder 18.

If remainder comes zero then the final dividend 18 will
be the required HCF.

$$\therefore \text{HCF} = 18$$

$$\begin{array}{r} 306 \overline{) 630} (2 \\ \underline{-612} \\ 18 306 (17 \\ \underline{-18} \\ 126 \\ \underline{-126} \\ 0 \end{array}$$

Pay Attention

Find the HCF of 671, 781 and 1441 by division methods.

$$(i) \quad 671 \overline{) 781} (1$$

$$\begin{array}{r} \underline{-671} \\ 110 671 (6 \\ \underline{-660} \\ 11 110 (10 \\ \underline{-11} \\ 0 \\ 0 \\ \underline{} \\ 0 \end{array}$$

$$(ii) \quad 11 \overline{) 1441} (131$$

$$\begin{array}{r} \underline{-11} \\ 34 \\ \underline{-33} \\ 11 \\ \underline{-11} \\ 0 \end{array}$$

$$\therefore \text{HCF of 671 and 781 is 11}$$

So, HCF of above three numbers is 11.

Note : We find HCF of third number with the HCF of any given two numbers, in case of more than two numbers. This same method is applied for next number also. The final HCF is the required HCF.

Example : Find HCF of 2052, 3996 and 7380 by division method.

Solution :

$$\begin{array}{r} 2052 \overline{) 3996} \quad (1 \\ -2052 \\ \hline \end{array}$$

$$\begin{array}{r} 1944 \overline{) 2052} \quad (1 \\ -1944 \\ \hline \end{array}$$

$$\begin{array}{r} 108 \overline{) 1944} \quad (18 \\ -108 \\ \hline \end{array}$$

$$864$$

$$-864$$

$$\hline 0$$

$$\begin{array}{r} 108 \overline{) 7380} \quad (68 \\ -648 \\ \hline \end{array}$$

$$900$$

$$-864$$

$$\begin{array}{r} 36 \overline{) 108} \quad (3 \\ -108 \\ \hline \end{array}$$

$$0$$

$$\hline 0$$

$\therefore$ HCF of all three numbers 2052, 3996 and 7380 = 36

Therefore HCF of 2052 and 3996 = 108

Discuss these and find conclusion

1. What will be HCF of two prime numbers ?
2. What will be HCF of two consecutive even numbers ?
3. Will HCF of any two even numbers be even or odd ?
4. Will HCF of one even and one odd number be even or odd ?
5. Find HCF of 8 and 12
6. Find HCF of 12 and 16
7. The HCF of two numbers is 6 then find, what will be remainder if first number is divided by 6 ?

Exercise 10(c)

1. Find HCF of the following numbers by prime factorisation.
 - (i) 144, 198
 - (ii) 225, 450
 - (iii) 13, 39, 273
 - (iv) 120, 144, 204
 - (v) 101, 909, 1111
 - (vi) 625, 3125, 15625
2. Find HCF of the following numbers by division method :
 - (i) 442, 1261
 - (ii) 935, 1320
 - (iii) 1624, 522, 1276
 - (iv) 2241, 8217, 747

3. Find the biggest number, by which 125 and 94 are divided gives remainder 1 in each case.
(Hint : Required HCF is the HCF of 124 and 93 which is obtained by subtracting 1 from each number)
4. Find the biggest number which divides 49, 59 and 109 gives remainder 1, 3 and 5 respectively.
5. Divide the numerator and denominator of following fractions by their HCF and convert it into the simplest form.

(i) $\frac{256}{1444}$

(ii) $\frac{286}{468}$

(iii) $\frac{6633}{15075}$

10.9 Multiple, common multiple and Least common multiple or LCM.

See the following table

Multiple of 8	6, 16, 24, 32, 40, 48, 56, 64, 72,
Multiple of 12	12, 24, 36, 48, 60, 72, 84, 96, 108.....
Common multiple of 8 and 12	24, 48, 72,
Least common multiple of 8 and 12	24

Here we see that common multiples of 8 and 12 are 24, 48, 72,,,,, but least common multiple is 24. Therefore, least common multiple of 8 and 12 is 24.

Activity :

- The teacher will say to the students to write the numbers 1 to 100.
Example—will say to give red colour on the multiples of 3. In similar way he will say to give black colour on the multiple of 7.
- Now LCM of (3, 7) will be the first number of the number chart on which red and black colours present.

The smallest common multiple of two or more than two given numbers is called the least common multiple of the given numbers.

Attempt it :

Write and fill up the following tables on your note book :

Multiples of 14

Multiples of 21

Common multiples of both

Least common multiple 14 and 21.

Conclusion

- Same multiples of the given numbers are called common multiples.
- The smallest common multiple of the common multiples of the number is the Least Common Multiple (LCM).
- Least common multiple of the numbers is completely divided by each of those numbers.

10.9.1. To find the least common multiple by prime factorisation

We are full aware with the method of factorisation for least common multiple, as :

$$16 = 2 \times 2 \times 2 \times 2$$

$$24 = 2 \times 2 \times 2 \times 3$$

$$\text{Therefore, LCM} = 2 \times 2 \times 2 \times 2 \times 3 = 48$$

In similar way,

$$36 = 2 \times 2 \times 3 \times 3$$

$$40 = 2 \times 2 \times 2 \times 5$$

$$84 = 2 \times 2 \times 3 \times 7$$

$$\text{Therefore LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 7 \times 5 = 2520$$

Give attention, for finding of least common multiple

1. First of all find the prime factor of the numbers.
2. Select the smallest factor to see the prime factors of all numbers. Write down that factor as highest times to present in any of these numbers.
3. After that select the bigger prime factor than previous factor. Write down this factor as highest times to present in any of these numbers.
4. In similar way, write down all the factors and find the LCM by multiply of these all factors.

The LCM can be find by the presenting the power of prime factors in the above examples, see

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$$\therefore \text{LCM} = 2^4 \times 3 = 48$$

and $36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$$40 = 2 \times 2 \times 2 \times 5 = 2^3 \times 5$$

$$84 = 2 \times 2 \times 3 \times 7 = 2^2 \times 3 \times 7$$

$$\therefore \text{LCM} = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

Attention please that we taken the highest powers of prime factors to find the LCM, it is not necessary that they are common in all.

10.9.2. To find the least common multiple of division method

Example 4 : Find least common multiple of 56, 70 and 84.

2	56, 70, 84
2	28, 35, 42
7	14, 35, 21
	2, 5, 3

Solution : Therefore, $\text{lcm} = 2 \times 2 \times 2 \times 7 \times 5 \times 3 = 840$.

Attempt it :

Find LCM of 100, 98 and 77.

Give attention, for finding of LCM by division method.

- First of all we divide all numbers by the smallest prime number which is common in at least two numbers and write down the quotient exactly below the numbers.
- Write down those numbers exactly below the numbers which are not dividend by that number.
- We repeat this process till the relatively prime numbers do not came below the given all numbers.
- We multiple all divisor numbers and the numbers which are relatively prime and present in the last row and find the required LCM.

Example 5 : Find LCM of 16, 32, and 64.

2	16,	32,	64
2	8,	16,	32
2	4,	8,	16
2	2,	4,	8
2	1,	2,	4
	1	1	2

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \\ = 64.$$

Example 6 : Find LCM of 8 and 15

Solution

First method :

$$8 = 2 \times 2 \times 2$$

$$15 = 3 \times 5$$

$$\therefore \text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \\ = 120$$

Second Method :

2	8,	15
2	4,	15
2	2,	15
3	1,	15
	1,	15

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \\ = 120$$

It is clear from the above examples.

- The LCM of two or more than two numbers is not less than any of them. It can be equal to biggest number of them.
- The LCM of any two relatively prime or prime numbers is equal to multiplication of these numbers.

Exercise 10(d)

1. Fill the appropriate number in the question mark

6	18	18
---	----	----

11	77	7
----	----	---

14	?	21
----	---	----

- (a) 34 (b) 42 (c) 24 (d) 28
2. Find out those common multiples of the following number-pairs whose value is less than 80
 (a) 9 and 15 (b) 6 and 10 (c) 8 and 9 (d) 7 and 11
3. Find out those numbers upto 60 which are completely divisible by 4 and 6 both.

4. Find out LCM of the following numbers.
 (a) 5, 10, 15 (b) 16, 44, 64 (c) 10, 65, 91
 (d) 22, 121, 418 (e) 14, 28, 35, 56.
5. Find out the smallest number which is completely divisible by 20, 25 and 40.
 (Hint : Required number is the lcm of given numbers)
6. Find out the smallest number in which if we add 3 then the sum is completely divisible by 36, 45 and 50 separately.
7. Find out the smallest number which gives remainder 3 in each case after divided by 75, 80 and 135.
8. Find out the smallest number from which 7 is subtracted, after that the remaining number is completely divisible by 20, 28, 35 and 105.
9. Find out the smallest number which gives remainder 17, 27, and 37 after divided by 35, 45, and 55 respectively.

[Hint : Note that difference of each divisor and its remainder is 18.

Therefore, required number = L.C.M. = 18]

10.10 Relation between highest common factor and least common multiple of two numbers :

Discuss and find the conclusion :

First number	Second number	HCF	LCM	HCF $\times$ LCM	First number $\times$ Second number
6	8	2	24	$2 \times 24 = 48$	$6 \times 8 = 48$
4	9	1	36	$1 \times 36 = 36$	$4 \times 9 = 36$
30	36	6	180	$6 \times 180 = 1080$	$30 \times 36 = 1080$
35	40	5	280	$5 \times 280 = 1400$	$35 \times 40 = 1400$

Here we see that in the above table, the product of both numbers is equal to the product of highest common factor and least common multiple of those.

Conclusion :

In the case of two numbers :

$$\text{HCF} \times \text{LCM} = \text{First number} \times \text{Second number}$$

$$\text{Therefore LCM of two numbers} = \frac{\text{Product of numbers}}{\text{their HCF}}$$

For finding LCM of two big numbers use of this formula is useful and simple.

Think and discuss :

Is this relation possible for three numbers ? Note that this relation is not true for LCM and HCF of more than two numbers. As : LCM of 5 15 and 20 = 60 and HCF = 5 but $5 \times 60 \neq 5 \times 15 \times 20$.

Example 7 : Find LCM of 117 and 221.

Solution : 117) 221 (1

-771

104) 117 (1

-104

13) 104 (8

-104

0

HCF = 13.

$$\begin{aligned}\therefore \text{LCM} &= \frac{\text{Product of numbers}}{\text{their HCF}} \\ &= \frac{117 \times 221}{13} = 1989\end{aligned}$$

Do it :

If HCF of two numbers is 18 and LCM is 504. If one number is 72 then find the second number.

We know that HCF of numbers completely divides each of these numbers. Similarly each number completely divides their LCM.. This gives the conclusion that :

HCF of two or more than two numbers is a divisor of their LCM and LCM is the multiple of their HCF.

Exercise 10(e)

1. To fill the accurate option in the blank :

15	5	25
12		18

- (a) 3 (b) 6 (c) 9 (d) 12
2. Show that product of LCM and HCF is equal to the product of the given numbers in the following each pair of numbers :
- (a) 14, 21 (b) 25, 65 (c) 32, 96
(d) 81, 135 (e) 15, 125.
3. If HCF of two numbers is 16 and their product is 6400. Find their LCM.
4. Can the HCF of two numbers is 14 and their LCM is 204 ? Give reasons in the favour of answer.
5. Find HCF of 2211 and 5025 by division method. Find LCM of these numbers on this basis.
6. How many times the HCF present in the LCM of the numbers 95, 285 and 399 ?
7. 17 is the largest number which completely divides 102 and 187. Find the smallest number on this basis which is completely divisible by these numbers.

10.11 Solution of the problems based on HCF and LCM.

There are many problems of daily life whose solution is based on HCF or LCM, such type of the following problems.

Example 8 : A room whose length is 30 decimeter and breadth is 25 decimeter has been covered by square tiles such that the surface of that room is fully covered by it. What will be the minimum number of such tiles where as any tiles is not broken.

It is necessary for the least number of square tiles is that the size of tiles is biggest/largest. It is possible when the size of the side of the square tiles is HCF of length and breadth of the room. It means it is a problem which is solved by use of the HCF.

Solution : $30 = 2 \times 3 \times 5$, $25 = 5 \times 5$

Therefore, $\text{HCF} = 5$

$\therefore$ Side of the square tile is 5 decimeter.

We know that to cover the whole surface of the room, the area of the total number of tiles is equal to the area of the room.

$$\begin{aligned}
 \text{Area of the surface} &= \text{length} \times \text{breadth} \\
 \therefore &= 30 \text{ decimeter} \times 25 \text{ decimeter} \\
 \text{Area of square tiles} &= 5 \text{ decimeter} \times 5 \text{ decimeter} \\
 \text{So, the minimum number of square tiles} &= \frac{30 \text{ decimeter} \times 25 \text{ decimeter}}{5 \text{ decimeter} \times 5 \text{ decimeter}} \\
 &= 6 \times 5 = 30
 \end{aligned}$$

Example 9: A sports trainer divides children into groups of 10, 12 or 15 in the play ground providing different sporting material to the children. Thus in any case does not leave any child. Tell them what the minimum number of children will be :

Solution :

2	10,	12,	15
3	5,	6,	15
5	5,	2,	5
	1,	2,	1

$$\begin{aligned}
 \therefore \text{LCM} &= 2 \times 3 \times 5 \times 2 \\
 &= 60
 \end{aligned}$$

Exercise 10(f)

Select the correct option from the option given below and fill the blanks :

1.

8	24	12
15		35

- (a) 45 (b) 65
(c) 105 (d) 100

2.

24	12	60
21		49

- (a) 14 (b) 7
(c) 21 (d) 28

3. There is 120 students in class 1 and 90 students in class 2 of a primary school. What is the number of students in each group after distributing these large number of student numbers ?
4. Find the smallest number of copies which are divided into different bundles of 3, 6, 12 and 15 but have divided into equal numbers.
5. There is a square mat to covering a field of 55m. long and 22m wide. Find out at least how many laying mats will be required of the same measurements.
6. Three planets rotate around a star in 200, 250 and 300 days respectively to make one round. If they are in the same and one direction of the star at a day then in how many days after they will be in the same situation ?
7. There are 125m, 220m and 275m cloths respectively in three bundles. Find the biggest scale of measurement. So that is measure the cloths completely.
8. There are six bells started ringing together. If they sit intervals 2, 4, 6, 8, 10 and 12 seconds respectively. How many times they ring together in 30 minutes ?
9. A girl sets the mangoes of a basket 4, 6 and 9 in each pile respectively, but one mango is left in the basket in every time. How many are there in the basket ?
[Hint : LCM = 36, we get least number of mangoes of the basket after adding 1 mango in the LCM]
10. The circumference of the length the four wheel are 50 cm, 6 cm, 90 cm and 100 cm respectively. At least how much distance will be four wheels along with the fill circle ?
11. A trader goes to Kanpur every fourth day while second trader goes every 10 days. If both of them were gone to Kanpur on 3rd January. So tell the next date when they will go back to Kanpur together ?

Efficiency Exercise – 10

1. Write the true/False statements from the following in your note book.
 - (i) The Product of the two number is less than the product of their LCM and HCF.
 - (ii) In any number is divisible by two numbers separately then that number is also divisible by product.
 - (iii) One composite and second prime numbers can be relatively prime.
 - (iv) Prime numbers are co prime also.
 - (v) If the unit digit of a number is odd then that number is divisible by 2.
 - (vi) 724 is completely divisible by 4.
 - (vii) If a number is divisible by 12 then it will be divisible by 3.
 - (viii) The LCM of the given number is not less than the biggest number among them.

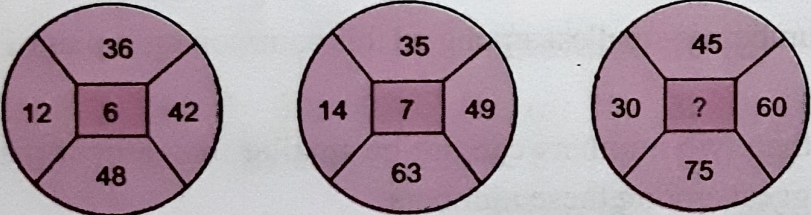
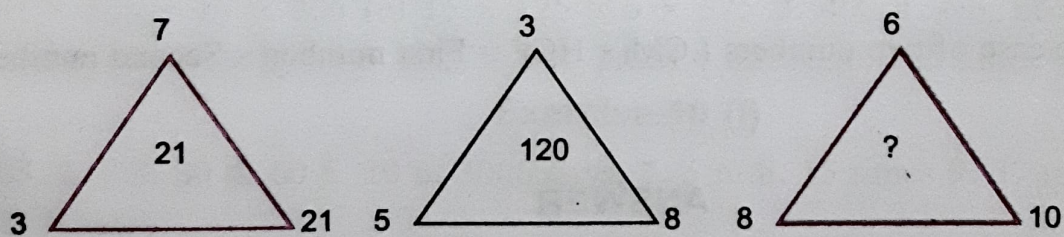
- (ix) The LCM of given numbers is not a multiple of it.
- (x) The HCF of co prime number is 1.
2. Write first five multiple of the following :
- (i) 13 (ii) 23 (iii) 126 (iv) 40
3. Of which 15 is a divisor of the following :
- (i) 3125 (ii) 122940 (iii) 151290
4. Find HCF of 5904 and 4848.
5. Four student run around a field. They complete the round in 30 second, 40 second, 50 seconds, and 60 seconds respectively. If they start to run together from any point of the field, then find the minimum time when they meet together at that point.
6. If HCF of two number is 35 and their LCM is 525. One number of them is 175 then which of the following will be the second number ?
- (i) 25 (ii) 49 (iii) 63 (iv) 105
7. Two tanks are filled with 72 liters and 116 liters of milk respectively. Then find a utensil of largest capacity in which the milk of both tanks could be fully measured.
8. The measurement of the surface of a room is $300\text{ cm} \times 425\text{ cm}$. Which measurement of the bigger tile can be applied so that the number of tiles is minimised ?
9. Three Hauges have 330, 375 and 450 litres of water respectively. Find largest capacity of pipa which can be used to get the water of these Hauge completely.
10. Find out the largest number from which to divide 804 and 1745 and gave remainder 5 and 6 respectively.
- [Hint : $804 - 5 = 799$ and $1745 - 6 = 1739$, the required number is HCF of these]
11. Find the largest number which divide 590, 908 and 1014 and give same remainder in each cases.
- (Hint : $908 - 590 = 318$ and $1014 - 908 = 106$. The HCF of these numbers is the required number).
12. Select the right option of the answer :
- (a) Three numbers are in the ratio 1 : 2 : 3. If their HCF is 12 then these numbers be ?
- (i) 12, 24, 36 (ii) 10, 20, 30 (iii) 5, 10, 15 (iv) 4, 8, 12
- (b) The lengths of three bomboos of wood are 36m, 45m and 63m respectively. We want to divide them into equal smaller pieces. Maximum length of each piece is ?
- (i) 9m (ii) 18m (iii) 51m (iv) 4.5 m

- (c) The length of three rods of measurement are 64 cm, 80 cm and 96 cm respectively. To use any one of the rod, to measure the which least length of cloth completely ?
 (i) 0.96 m (ii) 19.20 m (iii) 9.6 m (iv) 96 m
13. The traffic lights of three different cross roads change after 48 second, 72 seconds and 108 seconds simultaneously. If they change together at 8.20 hour then at what time they change together ?
14. The area of a rectangle is 56 cm^2 . What is the length and breadth of them in integer ?

SPECIAL QUESTION

1. A heap of coconut has been divided into group of 2, 3 and 5. One coconut remains in each cases least number of coconut in the heap is :
 (i) 31 (ii) 41 (iii) 51 (iv) 61

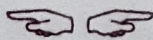
Instruction : Numbers are given in pictures with a special rule. Place of one number is blank which is shown by (?). Write the correct option from the given five option for the blanks:

2. 
- (i) 12 (ii) 15 (iii) 18 (iv) 21 (v) 24 Ans. 15
3. 
- (i) 8 (ii) 10 (iii) 21 (iv) 120 (v) 100 Ans. 120

We learn in this unit :

- How to identify the factor and multiples ?
 - Factor of a number divides that number completely.
 - Every number is a divisor of itself.
 - 1 is the divisor of each number.
 - The factor of any number is less than or equal to that number.

- (v) Each number is a multiple of its factor.
 - (vi) Every multiple of a number is greater than or equal to that number.
 - (vii) Each number is a multiple of itself.
2. How to identify the divisors of composite and prime numbers.
- (i) A number which has only two divisor 1 and the number itself, is called prime number. Those numbers which have more than two divisors are called divisible (Composite) numbers.
 - (ii) The common divisor of two numbers is only 1, are called co prime numbers,
 - (iii) If a number is divisible by the second number then it is also divisible by each of its divisors.
 - (iv) A number which is divided by two co prime numbers then it is also divisible by their product.
- 3.
- (i) The HCF of two or more numbers is the largest among all the divisor of the numbers.
 - (ii) The HCF of two or more than two numbers is equal to the smallest number or it is smaller than all of the numbers.
 - (iii) LCM of two or more numbers is smallest among all this common multiples of these numbers.
 - (iv) LCM of two or more than two numbers can not be smaller than any of them, It may be equal to the biggest among these numbers.
 - (v) LCM of any co prime numbers or prime numbers is equal to the product of the numbers.
 - (vi) In the case of two numbers $LCM \times HCF = \text{First number} \times \text{Second number}$.



ANSWER

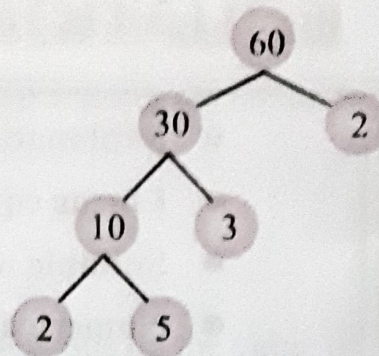
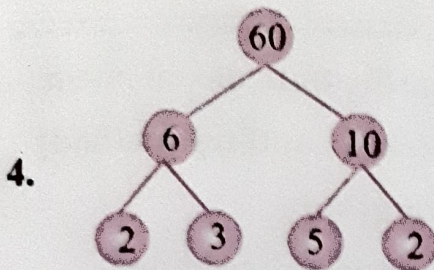
Exercise 10(a)

1. (i) c, (ii) e, (iii) d, (iv) 1, (v) b, 2. (a) 3, 6, 9, 12, 15, (b) 4, 8, 12, 16, 20, (c) 5, 10, 15, 20, 25 (d) 9, 18, 27, 36, 45, 3. (i) d, (ii) e, (iii) b, (iv) c, (v) a, 4. 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 5. 1, 2, 4, 8, 16, 31, 62, 124, 248, 496.

Sum of all factors = 992 (i.e., $2 \times 496 = 992$) it is two time of the number so it is a perfect number. 6. (i) divisor, self (ii) divisor, (iii) small, equal (iv) multiple, (v) 1, 2, 3, 6, 13, 26, 39, 48

Exercise 10 (b)

1. 3×3 , 2. 12, 3. $2 \times 2 \times 2 \times 3$



5. 9999, prime factors $3 \times 3 \times 11 \times 1101$, 6. 100000, $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
 7. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$, 8. $2 \times 2 \times 2 \times 2 \times 7$, $2 \times 2 \times 2 \times 3 \times 5$, $3 \times 3 \times 3 \times 5$, $2 \times 2 \times 5 \times 7$, $2 \times 3 \times 5 \times 5$, $2 \times 2 \times 307$
 9. (iv) 10. (i) divisible number (ii) prime number (iii) divisible, prime (iv) 7.

Exercise 10 (c)

1. (i) 18 (ii) 225 (iii) 13 (iv) 12 (v) 101 (vi) 625, 2. (i) 13 (ii) 55 (iii) 58 (iv) 747, 3. 31,
 4. 8 5. (i) $\frac{64}{361}$ (ii) $\frac{11}{18}$, (iii) $\frac{11}{25}$.

Exercise 10 (d)

1. 42, 2. (a) 45 (b) 30, 60, (c) 72, (d) 77, 3. 12, 24, 36, 48, 60, 4. (a) 30, (b) 704,
 (c) 910, (d) 4598, (e) 280, 5. 200 6. 897, 7. 10803, 8. 427, 9. 3447

Exercise 10 (e)

1. 6, 2. (a) $14 \times 21 = 7 \times 42$, (b) $25 \times 65 = 5 \times 325$ (c) $32 \times 96 = 32 \times 96$
 (d) $81 \times 135 = 27 \times 405$ (e) $15 \times 125 = 5 \times 375$ 3. 400 4. No, because, 204 is not
 completely divisible by 14 5. HCF = 201, LCM = 55275, 6. 105 times 7. 1122

Exercise 10 (f)

1. 105. 2. 7 3. 30 4. 60 5. 10 6. 3000 days, 7. 5 m 8. 15 times 9. 37 mangoes 10. 9 m
 11. 23 January

Efficiency Exercise 10

1. (i) False (ii) False (iii) True (iv) True (v) False (vi) True (vii) True (viii) True (ix) False (x)
 True. 2. (i) 13, 26, 39, 52, 65 (ii) 23, 46, 69, 92, 115 (ii) 26, 52, 78, 104, 130 (iv) 40, 80,
 120, 160, 200, 3. (ii) 122940 (iii) 151290 4. 48, 5. 600 seconds or 10 minutes, 6. (iv) 105
 7. 4 litres 8. 25 cm $\times$ 25 cm 9. 15 litre, 10. 47, 11. 106, 12. (a) (i) 12, 24, 36, (b) (i) 9 m
 (c) (ii) 9.60 m, 13. 8 hours 27 min. 12 sec., 14. 1 cm $\times$ 56 cm, 2 cm $\times$ 28 cm, 4 cm $\times$ 14
 cm, 7 cm $\times$ 8 cm.

