

UNIT-11 EQUATION : IN ONE VARIABLE



- Explanation of the meaning of mathematical statement.
- Linear equation in one variable (error and try method)
- Suitable method for solving of equation.
- Numerical questions of linear equations based on daily life.
- Alternative method for solving the equation.

11.1 Introduction :

In the previous chapter, we have studied method of algebraic expression, terms of the expression and the identification of equal and unequal terms. Simultaneously, the term described in words and sentences can be related to ordinary language of daily life. Let us understand the mathematical statement of ordinary language with an expression of the variable, by an example.

Rakesh said to Appu, “4 times as many flowers I have. [F] add 5 flowers then the total number of flowers becomes 45”. Write in the form of an algebraic expression.

Appu assumes that Rakesh has x flowers. Then 4 times the number of flowers $= 4x$. Now by adding 5 flowers, we get that the number of flowers $= 4x + 5$. This is an algebraic expression which is made of letter numbers and numbers and the fundamental operations. Therefore, according to the statement, $4x + 5 = 45$, which is a linear equation of one variable. In this unit we will study the linear equation (in one variable).

11.2 The Meaning of the Mathematical Statement

We speak many kinds of sentences in our daily life conversation. Read some of these following:

1. There will be rain in the night.
2. India will win today's match.
3. Delhi is the capital of India.
4. Number 6 is bigger than number 10.
5. Five is smaller than three.

It can not be said surely whether it is true or false for first and the second statement. Although it is sure that sentence 3, 4 and 5 may be true or false.

The sentence that is either true or false, is called statement.

Now we think about some other statements which are based on fundamental operations :

- | | |
|------------------------|--|
| (1) $6 + 5 = 11$ | (True) |
| (2) $6 > 4$ | (True) |
| (3) $3 < 2$ | (False) |
| (4) $x + 2 = 3$ | (Truthness depends on the value of x) |
| (5) $2x < 7$ | (Truthness depends on the value of x) |
| (6) $x - 6 = 9$ | (Truthness depends on the value of x) |

Here we see that the statement (1), (2) and (3) are those statements in which letter numbers are not present. These statements are called always true or always false.

Statements (4), (5) and (6) are not always true or false. Its truthness depends on the value of x .

For example :

$x + 2 = 3$, It is true for value of x is 1 and it is false for all the other values.

Again we think about statements (1), (4) (6)

- | | | |
|--------------|-------|-----|
| $6 + 5 = 11$ | | (1) |
| $x + 2 = 3$ | | (4) |
| $x - 6 = 9$ | | (6) |

The symbol '=', sign of equality is used in these statements. So, these statements are called statements of equality.

The truthness of statements (4) and (6) depend on the value of x .

11.3 What is the equation ?

Under the concept of algebraic expression, the letter V, N etc. are formed by match stick and to know the number of match stick used in it, a rule was found.

Note that the number of match stick is 2 for making a copy of 'V' but for n such copies we require $2n$ sticks whereas value of n can be 1, 2, 3, 4, ...

Similarly, we see that the number of the required match stick to create a copy of N is 3. But for creating n such copies of N, required number is $3n$, where value of n can be 1, 2, 3,

Here we see that the value of number of match sticks depends on the value of n . Therefore n is called the variable. To show the variables we use the English alphabets $l, m, n, p, q, r, x, y, z$, etc.

Try it :

Create a copy of M with the help of match sticks. For the creation of separate copies of M, find a formula for using of number of match sticks. In above, if you are given 10 match sticks then how many patterns of V can be formed ?

We have seen that for V, no. of required match stick is $2n$, where n , is the number of patterns of V.

Therefore $2n = 10$... (1)

Here we get a condition, which is fulfilled by values of n , we can check this by the following tables.

Number of V formed	1	2	3	4	5	6	7	...
Number of Necessary match sticks	2	4	6	8	10	12	14	...

Conversely, if you are given the number match sticks then we can find the number of copies of V.

If you are given 10 match sticks then how many V you can formed ? In this position, $2n = 10$, It is such a condition which is satisfied by variable n .

Value of n	Value of $2n$	Condition satisfied Yes/No
1	2	No
2	4	No
3	6	No
4	8	No
5	10	Yes
6	12	No

We find that only for $n = 5$, satisfied the condition $2n = 10$. It is not satisfied by any values of n , other than 5.

Conclusion

An equation is a condition of algebraic expression with equality sign, in which for a special value of variable, the value of both sides of the expression (inequality) is equal. This special value of the variable is called the solution of the equation.

An equation is a type of balanced scale, for which we put weight on the left plank and the weighing object on the right plank of the scale.

Change the equations to statements :

$$(i) x + 7 = 12, \quad (ii) 5x = 20, \quad (iii) \frac{m}{3} - 5 = 10$$

To solve the equation

$$3x + 7 = 28:$$

We use stepwise method for separating x on left side of the equation. Here, left side is $3x + 7$. We subtract 7 on both side for separating $3x$ so that the equation is balanced in each case.

Therefore, subtracting 7 on both sides :

$$3x + 7 - 7 = 28 - 7$$

$$3x = 21$$

Now both sides are divided by 3.

$$\frac{3x}{3} = \frac{21}{3}$$

$x = 7$ is the solution of the equation.

Formation of Equation

Shaili gave some ₹ to his brother Vishal and her mother also gave ₹ 5 to Vishal. Now he has ₹ 50. How many ₹ Shaili gave to Vishal ?

Suppose that Shaili gave ₹ x to Vishal and mother give ₹ 5 to Vishal. Now Vishal has ₹ 50.

$$\text{So, } x + 5 = 50$$

$$x = 50 - 5 = 45$$

Write down the following mathematical statements in algebraic expression. Check whether it is an equation or not .

(1) If you subtract 8 from 6 times of a number you get 10.

(2) If 5 is added to 4 times a number we get 21.

$$(1) 6x - 8 = 10$$

$$(2) 4x + 5 = 21$$

$$\text{Checking : } 6x - 8 = 10$$

Value of x	Left Side	Right Side
1	$6 \times 1 - 8 = -2$	10
2	$6 \times 2 - 8 = 4$	10
3	$6 \times 3 - 8 = 10$	10

The algebraic expression form of equation (1) is $6x - 8 = 10$. It is an equation whose solution is $x = 3$.

Checking : $4x + 5 = 21$

x	Left Side	Right Side
1	$4 \times 1 + 5 = 9$	21
2	$4 \times 2 + 5 = 13$	21
3	$4 \times 3 + 5 = 17$	21
4	$4 \times 4 + 5 = 21$	21

The algebraic expression form of equation (2) is $4x + 5 = 21$, It is an equation whose solution is $x = 4$.

Pay Attention

The sign of equality of the equation shows that for the special value of the variable, the value of the expressions in left hand side of sign of equality and the value of the expression on right hand side of the sign of equality are simultaneously equal.

If any sign other than equality between the left hand and right hand, then it is not an equation.

$6x - 8 > 10$	not an equation
$6x - 8 < 10$	not an equation
$6x - 8 = 10$	is an equation

The right hand side of the above equation is numerical which is an arithmetical expression. But it is always not necessary. Right hand side may be an expression with a variable also.

It is necessary that an expression with variable is present at least one of the side either left hand or right hand side, otherwise it will be not an equation, but it will be an arithmetic inequality.

When the maximum degree of the variable present in any equation is one then such type of equation is called a linear equation.

$6x - 8 = 10$ and $4x + 5 = 20$ are linear equations.

Write the following in form of mathematical statement and select linear equations :

1. $11 + 12 = 23$
2. $x + 4 = 6$
3. $10 \times 3 = 30$
4. $2x + 5 = x - 7$

 **Attempt these**

Separate the left hand side and right hand side of the following equations :

- | | |
|----------------------------|---------------------|
| 1. $x + 12 = 13$ | 2. $3x + 5 = 17$ |
| 3. $10x = 30$ | 4. $2x + 5 = x + 7$ |
| 5. $\frac{x}{3} = 14 - 2x$ | |

Solution of the equation

To think about the following equation :

$$8 + x = 13$$

This equation shows that if we add 8 in x then the sum is 13.

Here it is clear that value of x is 5, because 5 is added in 8 gives 13.

It can be also said that 5 is substituted at the place of x then value of left hand side is equal to the value of right hand side. That is equation is satisfied.

Therefore, the solution of the equation is $x = 5$. Also 5 is called root of the equation.

That number which is substituted at the place of variable, satisfies the equation, is called the solution of the equation.

 **Attempt it :**

Solve these :

- | | |
|----------------|----------------|
| 1. $x + 1 = 2$ | 2. $x + 7 = 5$ |
| 3. $x + 2 = 2$ | |

11.4 To solve the Linear Equation by Error and Try Method :

Example: $x + 6 = 9$

To solve this equation, that is to find the value of x , we substitute different values of x in the following table as equation $x + 6 = 9$.

Value of x	Value of left hand side ($x + 6$)	Give value of right hand side
0	$0 + 6 = 6$	9
1	$1 + 6 = 7$	9
2	$2 + 6 = 8$	9
3	$3 + 6 = 9$	9

We see that 3 is the only that value from the different values of x , which satisfies the equation.

That is $x = 3$ is the solution of the equation.

Example 2 : Solve the equation $x + 7 = 3$.

Equation $x + 7 = 3$

Different values of x	values of LHS ($x + 7$)	Given value of RHS
0	$0 + 7 = 7$	3
1	$1 + 7 = 8$	3
2	$2 + 7 = 9$	3
-1	$-1 + 7 = 6$	3
-2	$-2 + 7 = 5$	3
-3	$-3 + 7 = 4$	3
-4	$-4 + 7 = 3$	3

We see that if we put positive value of x in left hand side then it is regularly increased than value of right hand side. So, putting negative values of x , $x = -4$ satisfies the equation.

Therefore $x = -4$ is the solution of the equation.

 **Attempt it :**

Solve the following equations by error and try method (tabular method) :

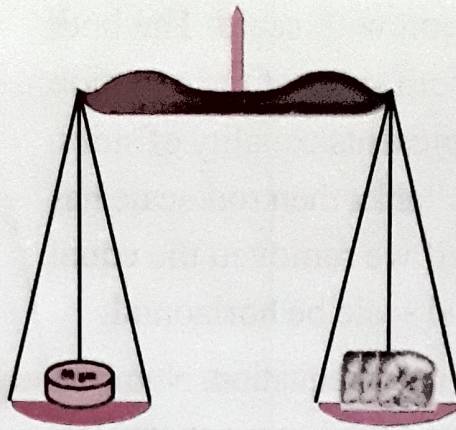
(i) $x + 2 = 5$

(ii) $3 = x + 2$

(iii) $x + 8 = 5$

Suitable/Accurate method to solve the linear equations :

We have seen that the weighing of the objects on a scale. The weight (unknown value) of any object putting on the scale can be found by putting the bat (known value) on the second back of the scale. It is shown that equal weight on both backs of scale if the beam is in horizontal.



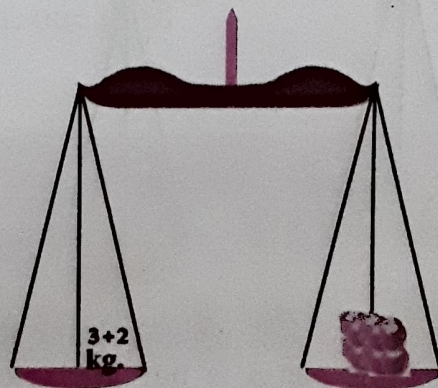
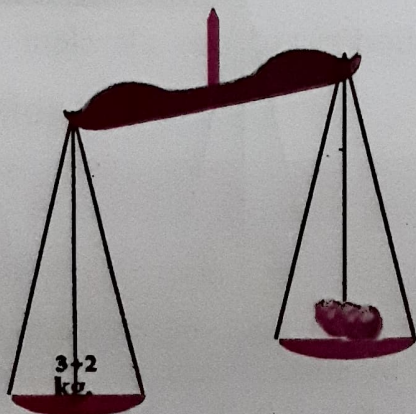
Unknown value (Object) = Known value (Bat)

We take an example to understand the equation.

Deepika has asked shopkeeper for 3 kg. of mangoes on a fruit shop. Shopkeeper put 3 kg. bat on the right Plank of the scale and put some mangoes on left Plank. The balance beam of scale was turned on the side of bat then shopkeeper has put one mango more on left back. The balance beam has been horizontal and both Plank are equal in balanced. At the same time her sister Sarika has been reached there and said for 2 kg. more mangoes together with this. The shopkeeper has put 2 kg. bat on the right Plank, the Plank turn down on that side. The shopkeeper starts to put the mangoes on left back, then the left Plank also start to turn to down and the Plank of bat starts to move up. The shopkeeper puts the mangoes of big size and small size by replacing to each other until the rod scale is not horizontal again. Shopkeeper says that take this 5 kg. mangoes.



Activity :



We can compare the equation with scale. The both Planks of the scale represent the both sides of the equation holding. Also, horizontal sides represents equality of sides. If we keep equal weights on both Planks then rod-scale has been horizontal. It is the same as if we removed the equal weight of weights after that the rod scale be horizontal.

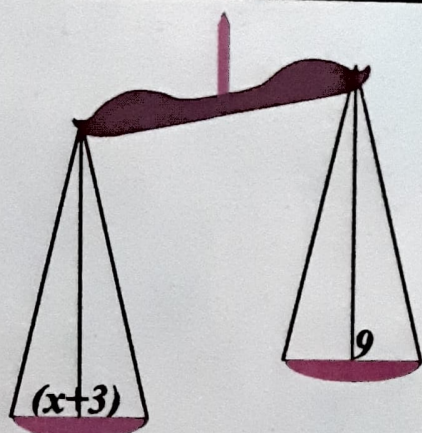
We use this principle in solving an equation. Numbers can be used to balance like bat. Let's take some examples,

Suppose that we put an object of x kg. on one back and a weight of 9 kg. weight on the second back, after that the balance beam of scale has been in horizontal.

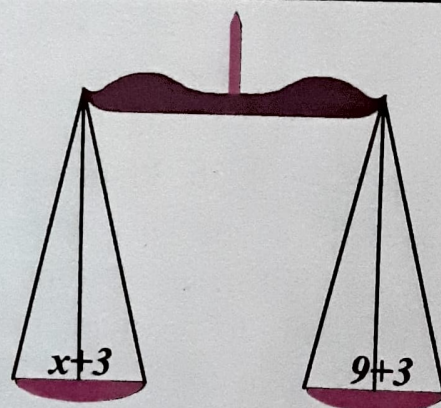
We see that balance beam is horizontal. That means the (value) weight of the objects on both backs are equal.

Activity

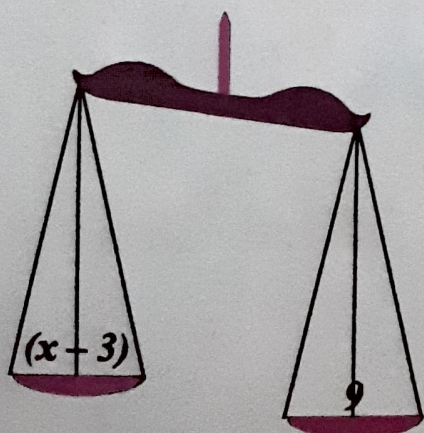
Now you see those activities which is written alongwith following figures.



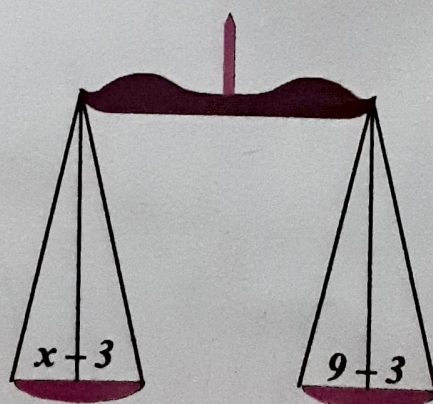
By putting three kg. on left Plank then beam is not horizontal



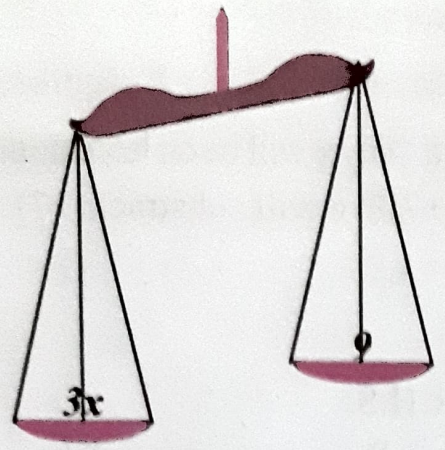
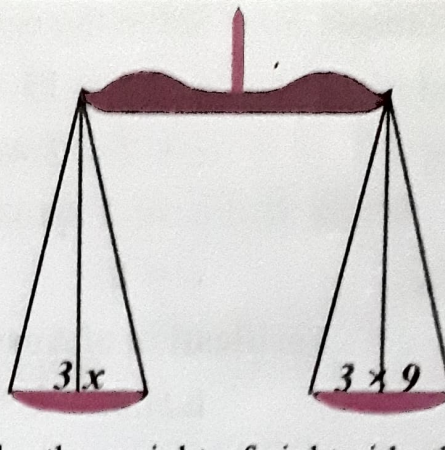
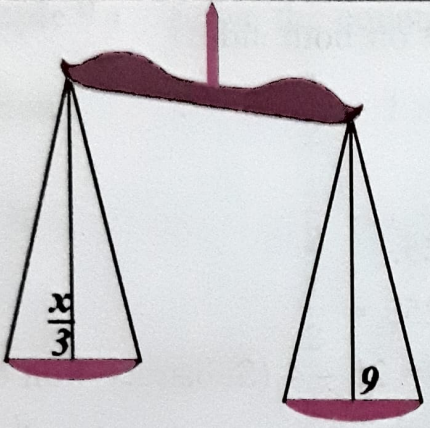
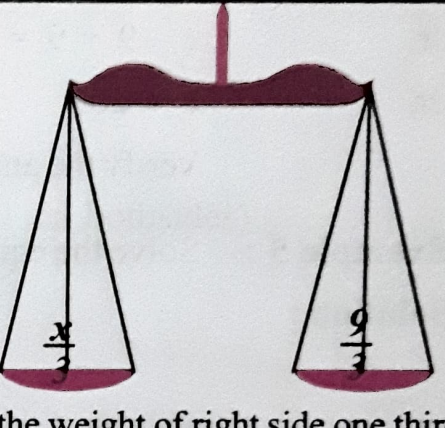
If put 3 kg. weight on right Plank to make beam horizontal then
 $x + 3 = 9 + 3$ or $x + 3 = 12$



If remove 3 kg. weight from left Plank then beam is not horizontal.



Remove 3 kg. weight from right Plank to make beam horizontal, then
 $x - 3 = 9 - 3$ or $x - 3 = 6$

 To make three times of the weight of left side then scale rod is not horizontal	 To make the weight of right side three times to make the scale rod horizontal $3x = 3 \times 9$ or $3x = 27$
 By making one third of the weight of left side, the scale rod is not horizontal.	 Make the weight of right side one third to make the scale rod horizontal that is : $\frac{x}{3} = \frac{9}{3}$

There are four facts come out from the above four activities :

The scale rod has been in horizontal in each case; if :

- To put equal weights on both pans.
- To remove equal weights from both pans.
- To make the weights of both pans double, triple ...
- To make the weights of both pans half, one third ...

✂ Conclusion

- The equation remains unchanged if we add, subtract multiply to a number or divided by a non-zero equal number on both sides of the equation.
- All these are called axioms. These are used in solving the equations.

Example 3 : Solve the equation $x + 7 = 15$

Solution : $x + 7 = 15$

Or, $x + 7 - 7 = 15 - 7$ [for solving it. Only x will be on left side therefore to remove (+7) we will subtract (+7)]

Or, $x = 8.$

Verification of Answer :

$$\text{L.H.S.} = x + 7 = 8 + 7 = 15 = \text{R.H.S.}$$

Example 4 : Solve the equation $x - 9 = 11.$

Solution : $x - 9 = 11$

Or, $x - 9 + 9 = 11 + 9$ (To add 9 on both sides)

Or, $x = 20$

Verify the answer by yourself.

Example 5 : Solve the equation $3n + 4 = 25.$

Solution : $3n + 4 = 25$

Or, $3n + 4 - 4 = 25 - 4$ (Subtract 4 on both sides)

Or, $3n = 21$

$$\frac{3n}{3} = \frac{21}{3} \text{ (divide both sides by 3)}$$

Therefore $x = 7$

Verify the answer yourself

Example 6 : Solve the equation $\frac{n}{3} = 20$

Solution : $\frac{n}{3} = 20$ or, $\frac{n}{3} \times 3 = 20 \times 3$ (multiply both sides by 3)

$$n = 60$$

Verify the answer yourself.

Example 7 : Solve the equation $5p = 15$

Solution : $5p = 15$

Or, $\frac{5p}{5} = \frac{15}{5} \dots$ (divide both sides by 5)

$$\text{Or, } p = 3$$

Verify the answer yourself.

Example 8 : Solve the equation $3a + 5 = 17$.

Solution : $3a + 5 = 17$

Or, $3a + 5 - 5 = 17 - 5$ (subtract 5 from both sides)

Or, $3a = 12$

$a = \frac{12}{3}$ (divide both sides by 3)

Or, $a = 4$

Verify the answer yourself.

Example 9 : Solve the equation $\frac{x}{2} - 1 = 2$.

Solution : $\frac{x}{2} - 1 = 2$

Or, $\frac{x}{2} - 1 + 1 = 2 + 1$ (add 1 on both sides)

$\frac{x}{2} = 3$

Or, $\frac{x \times 2}{2} = 3 \times 2$ (multiply 2 on both sides)

$x = 6$

Verify the answer yourself.

Example 10 : Solve the equation $7 + 5x = 15 + 3x$.

Solution : $7 + 5x = 15 + 3x$ (Variable may be any one of the side. We put the variation on left side for solving the problems)

or, $7 + 5x - 3x = 15 + 3x - 3x$ (subtract $3x$ on both sides)

or, $7 + 2x = 15$

or, $7 + 2x - 7 = 15 - 7$ (subtract 7 on both sides)

or, $2x = 8$

$\frac{2x}{2} = \frac{8}{2}$ (divide both sides by 2)

or, $x = 4$

therefore $x = 4$.

Verification of Answer :

L.H.S. $= 7 + 5x = 7 + 5 \times 4 = 7 + 20 = 27$

R.H.S. $= 15 + 3x = 15 + 3 \times 4 = 15 + 12 = 27$

Therefore L.H.S. = R.H.S.

Exercise 11 (a)

Solve the following equations and verify your answer :

1. (a) $x + 2 = 3$ (b) $3 + x = 4$ (c) $15 = a + 12$

2. (a) $11 = b - 3$ (b) $y - 6 = 0$ (c) $3 + g = 3$

3. (a) $5x = 10$ (b) $3x = 12$ (c) $33 = 11x$

4. (a) $\frac{x}{3} = 2$ (b) $\frac{7x}{15} = 0$ (c) $\frac{y}{5} = 1$

5. (a) $2x - 3 = 5$ (b) $3 + 4x = x + 9$

(c) $\frac{y}{7} + 1 = 2$ (d) $3 = 10 - \frac{z}{2}$

(e) $\frac{5y}{3} + 1 = 6$ (f) $1.2x + 3 = 4.2$

(g) $\frac{2}{3}x - 2 = 0$ (h) $\frac{2x-3}{5} = 7$

11.6 Numerical Questions of Linear Equations Based on Daily Life

11.6.1 To read the numerical questions related to linear equations and to form equations :

There is a boy in our village, his name is Sunil. He has a bag, there are some guavas in that. Farid has put 10 guavas in Sunil's bag. There are 22 guavas in the bag of this type. We can write this statement in a very simple language.

Suppose,

The number of guavas in Sunil's bag = x ... (1)

The number of guavas given by Farid = 10 ... (2)

Now, total number of guavas in Sunil's bag = $x + 10$... (3)

In bag, number of guavas according to the last line of equation = 22 ... (4)

Statement (3) and (4) are equal

So, we can write,

$$x + 10 = 22$$

Therefore, for giving the form of equation of any statement :

1. Read the question and search what to find . Suppose this unknown variable is x . Find an expression in the variable x according to the given question.
2. Read the question again. To establish a relation of equality between the expression of the variable and known quantities. The equation will be this.

Example 11 : The cricket match has been played between two teams of class 6. The runs of first team is 10 less than two times the runs of second team. If total runs of both teams are 110 then give the form of the equation of this condition.

Solution : Suppose that the no. of runs of second team = x

No. of runs of first team = $2x - 10$

Sum of runs of both teams = 110

According to the condition : $x + 2x - 10 = 110$

Or $3x - 10 = 110$

Attempt it :

Discuss on the following questions and write these in the form of the equation :

1. The sum of x and 6 is 13.
2. One number is 5 less than 9.
3. Two times of any number is 6.
4. One fifth part of a number is 3.

11.6.2. To find an unknown quantity in the questions related with daily life by the help of a linear equation.

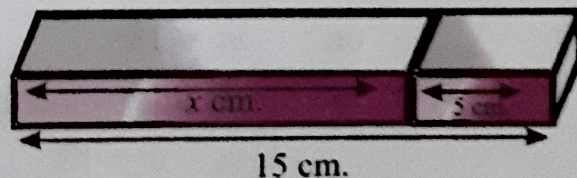
Example 12 : Form an equation from the given picture and find the value of x .

Solution :

$$x + 5 = 15$$

$$\text{Or } x + 5 - 5 = 15 - 5$$

$$\text{Or } x = 10.$$



Example 13 : The cost of 5 books is 14 less than the cost of 7 books. Find the cost of one book.

Solution : Suppose that cost of one book is ₹ x .

Cost of 5 books = ₹ $5x$

Cost of 7 books = ₹ $7x$.

According to the question, $5x = 7x - 14$

$$\text{or, } 5x - 7x = 7x - 14 - 7x$$

$$\text{or, } -2x = -14$$

$$\text{or, } 2x = 14$$

$$\text{or, } x = 7$$

Therefore cost of one book is ₹ 7.

Example 14 : Find the present age of Mohinder if he was 35 years before 10 years.

Solution : Suppose that present age of Mohinder is x year. Therefore, before 10 years his age will be $(x - 10)$ years.

According to the question,

$$x - 10 = 35$$

$$\text{or, } x - 10 + 10 = 35 + 10 \quad (10 \text{ is added on both sides})$$

$$\text{or, } x = 45$$

Therefore, present age of Mohinder = 45 years.

$$\text{or, } x = 45$$

Alteration method for solving the equation :

Come, Lets practice to solve some more equations. We will learn about the transpose of one number (term) (i.e., transferring of one side to another side) at the time of solving of these equations. You can transpose any number or term only in the place of adding or subtracting on both sides of the equation. Understand it with the help of the following example.

Example 1 : Solve $m - 6 = 12$

Solution : 6 is adding on both sides.

$$m - 6 + 6 = 12 + 6$$

$$\text{or, } m = 18$$

We can do this by the following method :

$$m - 6 = 12$$

$$m = 12 + 6 = 18$$

Here, we make -6 to +6 by transferring from left side to right side. This process is called alteration method.

Example 2 : Solve $P + 18 = 37$

Solution : by alteration $P = 37 - 18$
 $= 19$

You have seen that we add or subtract the same number on the both sides of the equation generally. The alteration of any number is same as the number to be added or subtracted on both sides for doing this we change the sign of that number. The rules used for numbers is same as for the expressions. To understand the alteration method by the following example.

To solve the equation by adding or subtracting a number on both sides	To solve the equation by alteration method
1. $7x - 10 = 5$ add 10 on both sides $7x - 10 + 10 = 5 + 10$ $7x = 15$	1. $7x - 10 = 5$ Alteration of (-10) from left side then sign will change i.e., $+10$ is for (-10) We seen $7x = 5 + 10$ Or $7x = 15$
2. $6P + 12 = 37$ Subtract 12 on both sides $6P + 12 - 12 = 37 - 12$ or, $6P = 25$	2. $6P + 12 = 37$. Alteration of $(+12)$ from left side, the sign will be changed i.e., (-12) will be for $(+12)$ $6P = 37 - 12$ $6P = 25$

To come, we solve some other problems by alteration method.

As, To solve the equations (a) $n + 3 = 36$ and (b) $x - 7 = 16$.

Solution : (a) $n + 3 = 36$

Transferring $(+3)$ from left side to right side

$$n = 36 - 3$$

$$n = 33$$

(b) $x - 7 = 16$

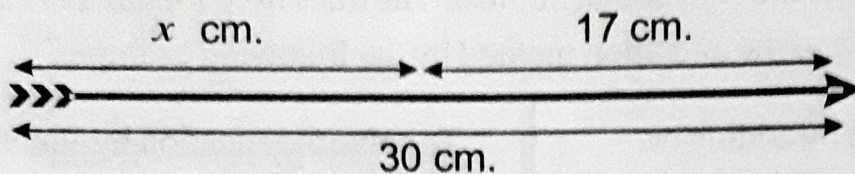
Transferring (-7) from left side to right side.

$$x = 16 + 7$$

$$x = 23.$$

Exercise 11 (b)

1. To form an equation with the help of the following figure, and find the value of x .

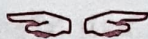


2. Age of Ram is 5 years more than Shyam, If the present age of Ram is 28 years, then find present age of Shyam.
3. After adding 5 in a number, we get 15. Find the number.
4. Shashi has bought some pencils. She gave 2 pencils to her sister. If three pencils remains to her now. Find, how many pencils she has bought ?
5. The weight of pragya became 17 kg. by increasing of 3 kg. from previous weight. What was the previous weight of her ?
6. A y meter long ribbon has divided into seven equal parts. If length of each part is 3 meter then find the value of y .
7. If half of a number is 10 greater than one fourth of that. Find that number.
8. There are 550 oranges on a trolley. Divide these oranges such as one trolley has 50 oranges more than the oranges of the second trolley.
9. Solve the following equations by error and try method :
- (a) $x + 6 = 10$ (b) $x + 9 = 5$
10. Solve the following equations by alteration method :
- (a) $P + 19 = 21$ (b) $n - 7 = 8$
- (c) $x - 11 = 20$ (d) $7 = 6 + 8$
- (e) $x + 5 = 0$

We Learnt in this Unit :

1. Variable a number whose value is not constant or stable.
2. An equation is condition on the variable we write an expression of variable and this equal to constant number.
as : $x - 4 = 1$, $4x = 12$, $5 - 2x = 3$ etc.
3. The left hand side and the right hand side are equal in an equation. There is an equal ($=$) sign between these two sides.
4. The equation which is satisfied by the fixed value of the variable, that value is called the solution of the equation.

5. To learn the error and trial method for solving the linear equation, we check if the equation is satisfied or not by the given values of the variable in this method. We put the different values of the variable in the equation till the equation is not satisfied. The equation satisfied by the value is called the solution of that equation.
6. We find only the variable on one side by adding, subtracting, multiplying or dividing both sides of the equation by an exact (accurate) number, and we find constant number on the other side. This constant number with respect to the variable is called the solution of the equation.
7. It is also taught that to solve the equations by alteration method. In this method we transfer the variable or constant from one side to another side. We change the sign '+', '-' by '-', '+' respectively in this process.
8. We taught the formation of equation and solving them of the numerical questions based on daily life.



ANSWERS

Exercise 11 (a)

1. (a) $x = 1$, (b) $x = 1$, (c) $a = 3$, 2. (a) $b = 14$, (b) $y = 6$, (c) $g = 0$, 3. (a) $g = 2$, (b) $x = 4$, (c) $x = 3$, 4. (a) $x = 6$, (b) $x = 0$, (c) $y = 5$, 5. (a) $x = 4$, (b) $x = 2$, (c) $y = 7$, (d) $z = 14$, (e) $y = 3$, (f) $x = 1$, (g) $x = 3$, (h) $x = 19$.

Exercise 11 (a)

1. (1) $x = 13$, (2) 23 years, (3) 10, (4) 5, (5) 14 kg. (6) 21 m., (7) 40 ; (8) 300, 250; (9) (a) 4, (b) -4; (10) (a) 2, (b) 15, (c) 31, (d) -1, (e) -5.

