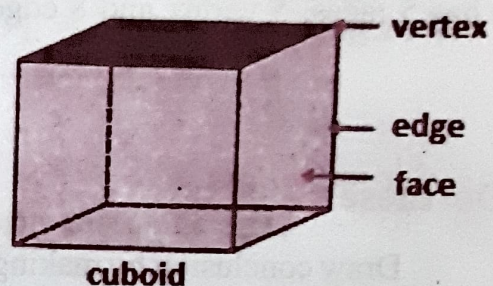




- Parts of Cube, Cuboid, Pyramid and Prism
- Concept of Volume
- Volume of Cube and Cuboid (Using formula)

16.1 Introduction:

You have studied about the solid objects and their dimensions under the geometrical concepts. You also know that some objects have a flat face, some have flat as well as curved and some have only curved faces. You have observed the cube, cuboid, cylinder, sphere and cone under the mathematical kit. All faces of cube and cuboid are planes, cylinder has both the ends planes and curved part at the middle, cone has one end planar and other end is pointed, called vertex, middle part is curved whereas sphere has only curved surface. Let us know about the edge, face and vertex of the objects that have same shape as that of cube and cuboid.



16.2 Parts of Cube, Cuboid, Pyramid and Prism

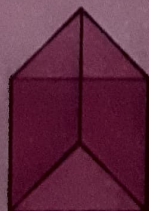
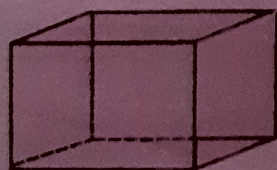
Do it, Think and Draw Conclusions:

Look carefully at the picture of the cuboid and tell the number of vertex, edge and faces.

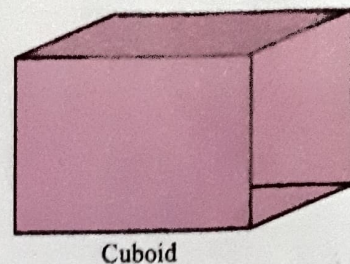
We see in the picture that there are 8 vertex, 12 edges and 6 faces in the cuboid. The vertex, edge and face are denoted by V, E and F respectively.

☞ Try it :

Calculate the number of vertex, edge and faces of the cube, cuboid and prism shown in the figure.

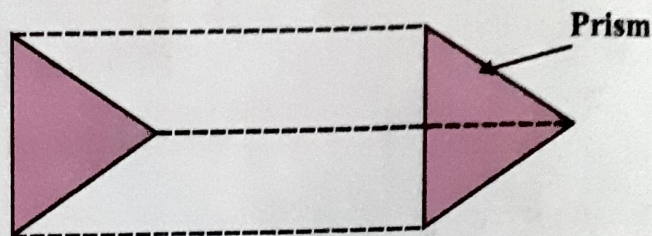


We noticed that the objects similar to the shape of cube or cuboid have 6 faces, 8 vertex and 12 edges.

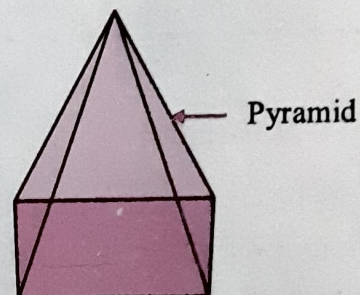


Cuboid

It is clear from the figure that the following prism has two triangular and three rectangular faces. Hence, it has 5 faces, 6 vertex and 9 edges.



We see in the picture that pyramid has 4 triangular and one rectangular face. In this way, it has 5 faces, 5 vertex and 8 edges.



Do these :

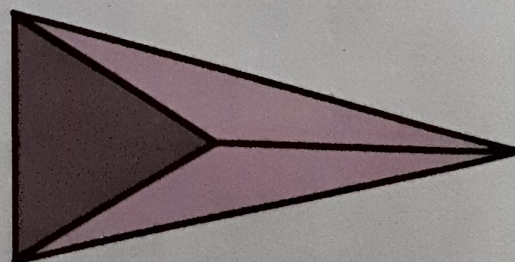
Draw conclusion by making a table for faces, vertex and edges of above cuboid, prism and pyramid.

Name	No. of Vertex (V)	No. of Faces (F)	No. of Edges (E)
Cuboid	8	6	12
Prism	6	5	9
Pyramid	5	5	8

We see from the above table that in each case, the sum of no. of vertex and no. of faces is 2 more than no. of edges. Thus, we can conclude that, $V+F=E+2$. This is called Euler's Formula.

Pay Attention:

If the base of the Pyramid is a triangle then this type of pyramid is called tetrahedral, because it has four faces. The base of the pyramid can be any polygon.



The relation of $V + F = E + 2$ is true for each of cube, cuboid, prism and pyramid.

Exercise 16 (a)

1. Identify the object that has same shape as that of cuboid:

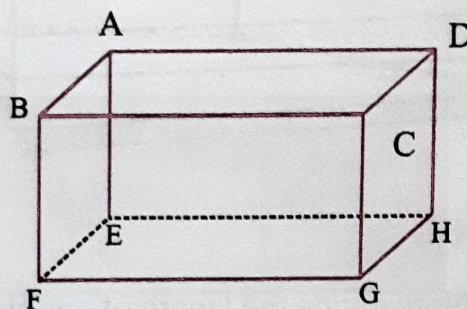
- | | |
|-------------------|-------------|
| (i) Ball | (ii) Box |
| (iii) Road roller | (iv) Funnel |
| (v) Almirah | (v) Book |
| (vii) Brick | |

2. (i) Write the name of edges that meets at point E in the adjacent figure.

(ii) Tell the name of the face parallel to the face ABCD in the adjacent figure.

3. Tell the number of vertex, faces and edge in a cubical dice. Is the relation $V + F = E + 2$ is true for this?

4. Write the sum of the number of points drawn on opposite faces of a dice.



16.3 Concept of Volume

The physical quantity that measures the space occupied by the object is called volume of that object.

Just as the unit of length is cm or meter, unit of area is square cm or square meter, in the same way the unit of volume is cubic cm or cubic meter.

The space occupied by a cube, in which length of each edge is 1 cm, is represented by 1 cubic cm.

i.e., 1 cubic cm volume is the volume of a cube, in which length of each edge is 1 cm.

☞ Try it :

What do you mean by 1 cubic meter volume?

Important :

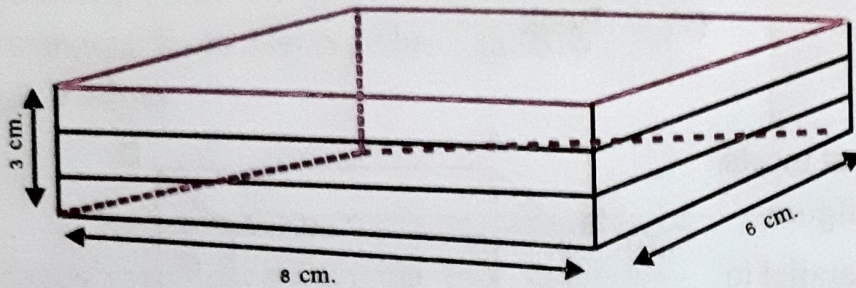
$$\begin{aligned} 1 \text{ cubic meter} &= 1 \text{ meter} \times 1 \text{ meter} \times 1 \text{ meter} \\ &= 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} \\ &= 1000000 \text{ cubic cm} = 10^6 \text{ cubic cm} \end{aligned}$$

Cubic cm is also written as cm^3 , similarly, cubic meter is also written as m^3 .

16.4 Volume of Cuboid and Cube (Using Formula)

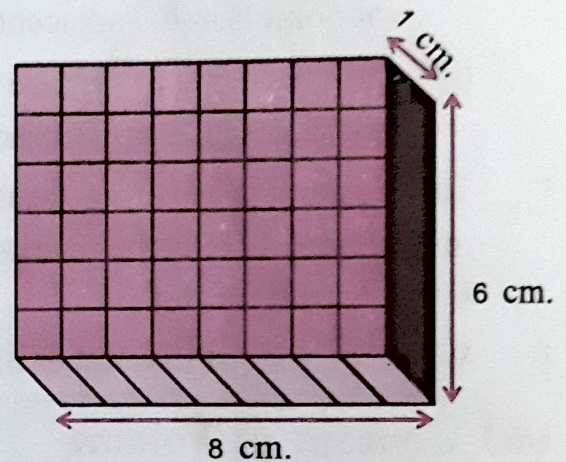
Think it, Discuss and Draw Conclusions:

To find the volume of a cuboidal soap of length 8 cm, breadth 6 cm and height 3 cm. Take a piece of soap in shape of cuboid of length 8 cm, breadth 6 cm and height 3 cm. Divide its height in 3 equal parts. Cut them with a sharp knife and separate them in 3 equal strips.



Activity

Now divide the length of one strip into 8 equal strips. Divide each of these 8 strips into 6 equal parts. Count the number of pieces of cubic cm obtained in this way.



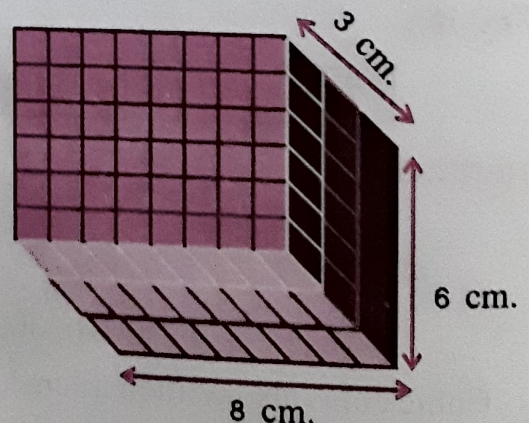
Try it :

1. How many pieces are obtained from one strip?
2. What are the length, breadth and height of each piece?
3. Is the volume of each piece in the form of a cube?
4. How many strips will obtain by cutting the soap?

Now we will see that there are total 48 pieces and each piece have length, breadth and height 1cm and each piece is cubical.

Can you Answer :

The volume of one strip will be equal to volume of how many cubes of side 1 cm. The volume of a strip is equal to the volume of all 48 cubical pieces obtained i.e. the volume of a strip is equal to 48 cubic cm.



If we put all three strip one above the other, then we again obtain the piece of soap as shown in the adjacent type.

Thus,

$$\begin{aligned}
 \text{Volume of the soap} &= 3 \times \text{Volume of a strip} \\
 &= 3 \times 48 \text{ cubic cm} \\
 &= 3 \times (6 \times 8) \text{ cubic cm} \\
 &= 3 \times 6 \times 8 \text{ cubic cm} \\
 &= \text{length} \times \text{breadth} \times \text{height}
 \end{aligned}$$

Therefore, we come to the conclusion that,

$$\begin{aligned}
 \text{Volume of a cuboid} &= \text{length} \times \text{breadth} \times \text{height} \\
 &= l \times b \times h \\
 \text{where } l &= \text{length, } b = \text{breadth, } h = \text{height.}
 \end{aligned}$$

We can find the volume of any soap or a cuboid using this formula, without cutting it.

Hence, The volume of the piece of soap = $8 \times 6 \times 3 \text{ cm}^3 = 144 \text{ cm}^3$.

Formula for the volume of a cube,

We know that, cube is a cuboid whose length, breadth and height are mutually equal. Therefore, we can get the formula to find the volume of the cube by substituting side of the cube **a** in place of **l**, **b** and **h** in the formula for volume of the cuboid.

Thus, we come to the conclusion that:

$$\begin{aligned}
 \text{Volume of the cube} &= a \times a \times a = a^3 \\
 \text{where } a &\text{ represents any side of the cube.}
 \end{aligned}$$

Example 1: Find the volume of the cuboid, whose length is 5.5 cm, breadth is 3.5 cm, height is 4.0 cm.

Solution: Length of the cuboid $l = 5.5 \text{ cm}$
 Breadth of cuboid $b = 3.5 \text{ cm}$
 Height of cuboid $h = 4.0 \text{ cm}$
 Volume of the cuboid $= l \times b \times h$
 $\therefore$ Volume of the cuboid $= 5.5 \times 3.5 \times 4.0 \text{ cm}^3$
 $= 77 \text{ cm}^3$
 Volume of the cuboid $= 77 \text{ cm}^3$

Example 2 : Each side of a cube is 6 m. Find the volume of this cube.

Solution: Each side of the cube $a = 6$ m
Volume of the cube $= a^3$
 $\therefore$ Volume of the cube $= 6^3 \text{ m}^3$
 $= 6 \times 6 \times 6 \text{ m}^3$
 $= 216 \text{ m}^3$
 $\therefore$ Volume of the cube $= 216 \text{ m}^3$.

Example 3: Find the volume of 8 m long, 3.5 m high and 20 cm wide wall.

Solution: length of the wall $= 8$ m
breadth of wall $= 3.5$ m
height of wall $= 20 \text{ cm} = 0.20 \text{ m}$
Volume of the wall $= l \times b \times h$
 $= 8 \times 3.5 \times 0.20 \text{ m}^3$
 $= 5.600 \text{ m}^3$
Volume of the wall $= 5.6 \text{ m}^3$

Example 4 : The length, breadth and height of a piece of wood in the shape of cuboid are 64 cm, 32 cm and 48 cm respectively. How many wooden plank of 16 cm length, 8 cm breadth and 12 cm height can be made from it?

Solution: length of the piece of wood $= 64$ cm
breadth of piece of wood $= 32$ cm
height of piece of wood $= 48$ cm
 $\therefore$ Volume of the piece of wood $= 64 \times 32 \times 48 \text{ cm}^3$
Similarly, volume of wooden plank $= 16 \times 8 \times 12 \text{ cm}^3$
 $\therefore$ No. of wooden plank that can
be made from wood piece $= \frac{\text{Volume of the piece of wood}}{\text{Volume of wooden plank}}$
 $= \frac{64 \times 32 \times 48 \text{ cm}^3}{16 \times 8 \times 12 \text{ cm}^3}$
No. of wooden plank $= 64$

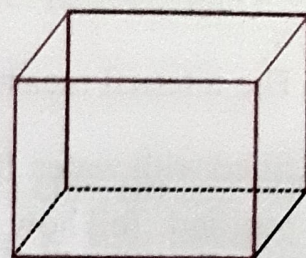
Know it as well:

The side of a cubical pot is 10 cm i.e. 1 dcm. What is its' volume?

$$\begin{aligned}\text{Volume of the cubical pot} &= 10 \times 10 \times 10 \text{ cm}^3 \\ &= 1000 \text{ cm}^3 (1 \text{ dcm}^3) \\ &= 1 \text{ litre}\end{aligned}$$

Similarly,

$$\begin{aligned}1 \text{ m}^3 &= 1000000 \text{ cm}^3 \\ &= 1000 \text{ litre.}\end{aligned}$$



EXERCISE 16 (b)

- Find the volume of the cuboid whose length, breadth and height are as follows:
 - Length = 8 cm, breadth = 5 cm, height = 4 cm.
 - Length = 80 cm, breadth = 40 cm, height = 1 m 20 cm.
 - Length = 14 cm, breadth = 8.5 cm, height = 5 cm.
 - Length = 1.4 cm, breadth = 0.5 cm, height = 0.4 m.
- Find the volume of the cube whose side is as follows:
 - Side = 12 cm
 - Side = 6.4 cm
 - Side = 7.2 cm.
 - Side = 1.3 cm.
- The length, breadth and height of a room are 5 m, 4 m and 3.5 m respectively. Find the volume of the room.
- The sides of two cubical objects are 2 cm and 4 cm respectively. Find the relation (ratio) between their volume V_1 and V_2 .
- The length and breadth of a field are 40 m and 15 m respectively. If a rainfall, of 50 m.m. has occurred on this field then check how many litres of water is dropped on the field.
- The length, breadth and height of a brick are 25 cm, 10 cm and 7.5 cm respectively. How many bricks will be used to build a wall of 5 m long, 3.5 m tall and 33 cm wide?
- The volume of a cuboidal piece of wood is 264 cm^3 . Find its height if the wood is 8 cm long and 6 cm wide.
- The length, breadth and height of a piece of wood in the shape of cuboid are 75 cm, 15 cm and 5 cm respectively. How many wooden plank of 15 cm length, 3 cm breadth and 1 cm height can be made from it?

9. A pit 5 m long, 3 m wide and 1.5 m deep is dug in a residential complex at raw land for ground water storage during the rainy season for ground water conservation. How many litres of water can be collected maximum in that pit?
10. The internal measures of a cuboidal tank are 5 m, 4 m and 3 m. $\frac{9}{10}$ part of the tank is filled with water. It cost Rs. 10 per 1000 litre of water to make water pollution free and purified. Tell how much money will be spent on purging the entire water of the tank.

Project:

Take a tea box in the cuboidal shape, whose length = 15 cm, breadth = 9 cm and height = 6 cm. Find the areas of all faces by opening it.

- (i) What is area of all faces of the box?
- (ii) Find the formula for total surface area, if the length = l cm, breadth = b cm and height = h cm of the box.

We learnt in this unit:

1. The vertex, edge and face of a solid geometrical object are usually denoted by V, E and F respectively.
2. The cuboid have 6 faces, 8 vertex and 12 edges.
3. The prism (triangular) has 5 faces, 6 vertex and 9 edges.
4. The pyramid (rectangular base) has 5 faces, 5 vertex and 8 edges.
5. The relation $V + F = E + 2$ is true for each of cube, cuboid, prism and pyramid.
6. The measure of the space occupied by an object is done by the volume and its unit is cubic unit (cubic cm or cubic m).

$$\begin{aligned} \text{7. Volume of a cuboid} &= \text{length} \times \text{breadth} \times \text{height} \\ &= l \times b \times h \end{aligned}$$

where, l represents length, b represents breadth, h represents height.

$$\begin{aligned} \text{8. Volume of the cube} &= \text{side} \times \text{side} \times \text{side} \\ &= a \times a \times a \text{ cubic unit} \end{aligned}$$

where a side of the cube.

Answers

Exercise 16 (a)

1. Box, Almirah, Book, Brick; 2. (i) EH, EA, EF; (ii) EFGE; 3. 6 face, 12 edges and 8 vertex, True; 4. 7

Exercise 16 (b)

1. (i) 160 cm^3 , (ii) 384000 cm^3 , (iii) 595 cm^3 , (iv) 28 cm^3 ; 2. (i) 1728 cm^3 , (ii) 262.144 cm^3 , (iii) 373.248 cm^3 , (iv) 2.197 cm^3 ; 3. 70 cm^3 ; 4. 1:8 ; 5. 30000 litre; 6. 3080 bricks; 7. 5.5 cm; 8. 125; 9. 22500 litre; 10. Rs. 540.

