



- Concept of Integers
- Absolute (Numerical) value of an Integer
- Basic operations on Integers
- Uses and types of Brackets
- Representation of Integers on Number Line
- BODMAS Rule

3.1 Introduction

You have studied about natural numbers, whole numbers and basic operations (Addition, Subtraction, multiplication and Division) of these numbers. Whole numbers were expanded to subtract large whole numbers from the small whole numbers. For this corresponding to natural numbers 1, 2, 3, corresponding numbers $-1, -2, -3$ are formed. Thus the expanded collection of whole Numbers is called the collection of Integers. In this unit we will learn about integers in detail.

3.2 Need for Integers

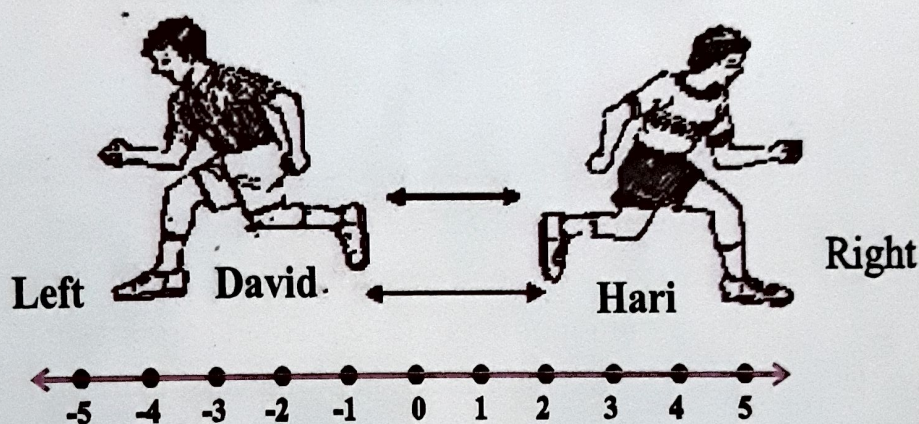
Mohan goes to the market to purchase a pen. He wants to purchase a pen costing ₹ 12 from his known shop but he has only Rs. 10 with him. He told the shopkeeper that he could not purchase the pen due to insufficient money. The Shopkeeper says to Mohan giving the pen to give the money what you have, and pay the remaining rupees later. Mohan was delighted, he took that pen and gave ₹ 10 to the shopkeeper. Tell, how much more money he needs to pay to the shopkeeper? Clearly, Mohan has to pay ₹ 2 more to the shopkeeper. The shopkeeper writes ₹ 2 in his diary as due amount from Mohan. The shopkeepers will write these ₹ 2 in the name of Mohan in his diary when he will return ₹ 2 of the credit to the shopkeeper. This type of situation occur in our daily lives . What is the sign with which the amount of borrowing of ₹ 2 is mentioned. We don't only lend money, but sometimes we have to borrow goods as well. Even when farmers sow seeds in the fields, sometimes, they borrow from their neighbour or a village person who gives the farmers the seeds to sow and return it when the crop is ready. In business sometimes trader gets the profit and sometimes there is a loss. In the modern multy-stories buildings, the storey are above the ground floor and are also built below the ground floor. Therefore, staircases are made both above and below ground floor if a person stepdown 5 stairs below the ground floor, how will we mark it ?

Mathematicians have given the concept of ‘–’, that is role of minus sign, for clarity in the above examples, the amount borrowed, the number of stairs descending from the ground floor etc. Since profit and loss are opposite situations and if profit is represented by ‘+’ sign loss can be represented by ‘–’ sign. Similarly temperature above 0°C is denoted as ‘+’ sign and temperature below 0°C is denoted by ‘–’ sign. On a number line distance to the right of zero is represented by ‘+’ sign and to the left of zero represented by ‘–’ sign.

Integers

During the study of whole numbers we saw that whole numbers do not obey commutative law for subtraction because there is no place of numbers with negative sign in the set of whole number. But in our daily life we need to have negative numbers as well as whole numbers. Let us understand this by an example.

Oppositeness



Suppose Hari and David have started walking from zero position in opposite directions. Hari moves the steps to the right of zero and David moves equal steps to the left of zero. If Hari moves 5 steps to the right of zero, David moves 5 steps to the left of zero.

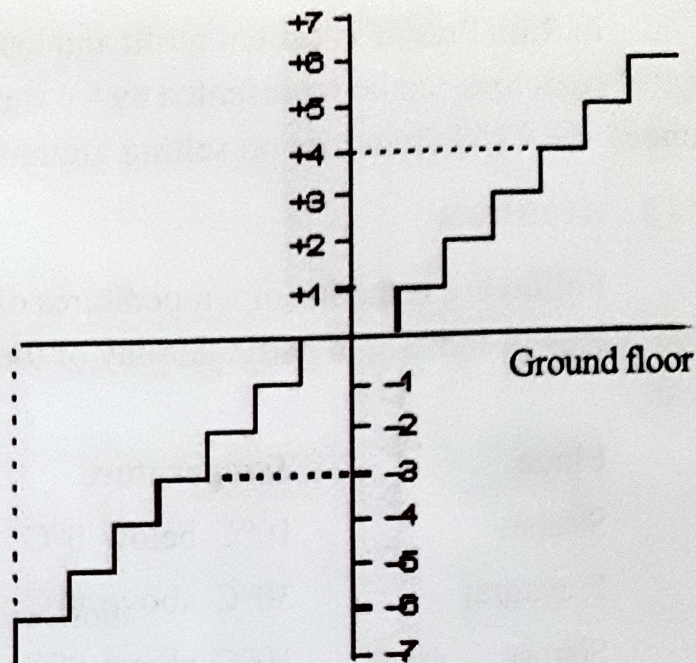
Moving of 5 towards right and 5 steps towards left represent the oppositeness. Let the steps to the right of zero be represented by ‘+’ sign and to the left of zero represented by ‘–’ sign. In this way position of Hari and David will be represented by +5 and –5 respectively.

Try these:

Represent the following positions with sign

- (a) 8 steps to the left of zero
- (b) 7 steps to the right of zero
- (c) 11 steps to the right of zero
- (d) 6 steps to the left of zero

2. In Majeed's house there are stairs for going up to the terrace and for going down to the godown. It is evident from the figure that the distance above the ground floor is taken stairs going upto the terrace as positive distance and the distance below the ground floor as negative distance.



Try these:

Tell the following position with sign :

- Going 4 steps up from the ground floor.
- Going 3 steps down from the ground floor.
- Going 5 steps up from the ground floor and then going 2 steps up further from there.
- Going 3 steps down from the ground floor and then going down further 4 steps from there.
- Going down 4 steps from the ground floor and then moving up 15 steps from there.
- Going 5 steps down from the ground floor and then going down further 3 steps from there. Let us consider that the position above from the ground floor be represented by '+' sign and below the ground floor be represented by '-' sign.

From the above activity position in first condition = 4 steps up stairs = +4

Position in second condition = 3 steps down from the ground floor = -3

Write the position in other conditions :

- | | |
|--------|--------|
| (a) +4 | (b) -3 |
| (c) | (d) |
| (e) | (f) |

Think

Following is an account of a shopkeeper Ram Prasad which shows profit and loss from the sale of certain items in a day.

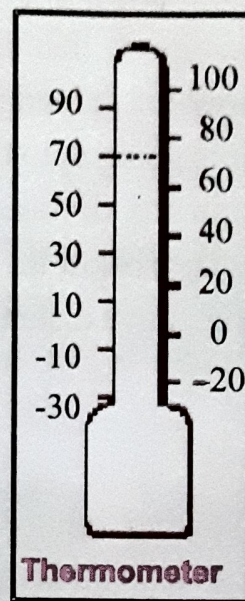
Name of items	Profit	Loss
1. Mustard Oil	₹ 350.00	-
2. Groundnut Oil	-	₹ 120.00
3. Black Papper	₹ 225.00	-
4. Turmeric	₹ 225.00	-
5. Wheat flour	-	₹ 425.00
6. Rice	₹ 321.00	-

In Ram Prasad's account profit and loss are opposite situation. If profit is represented by '+' sign, loss can be represented by '-' sign. Therefore if profit on selling mustard oil, in rupees = + ₹350 then loss on selling groundnut oil in rupees = - ₹ 120.

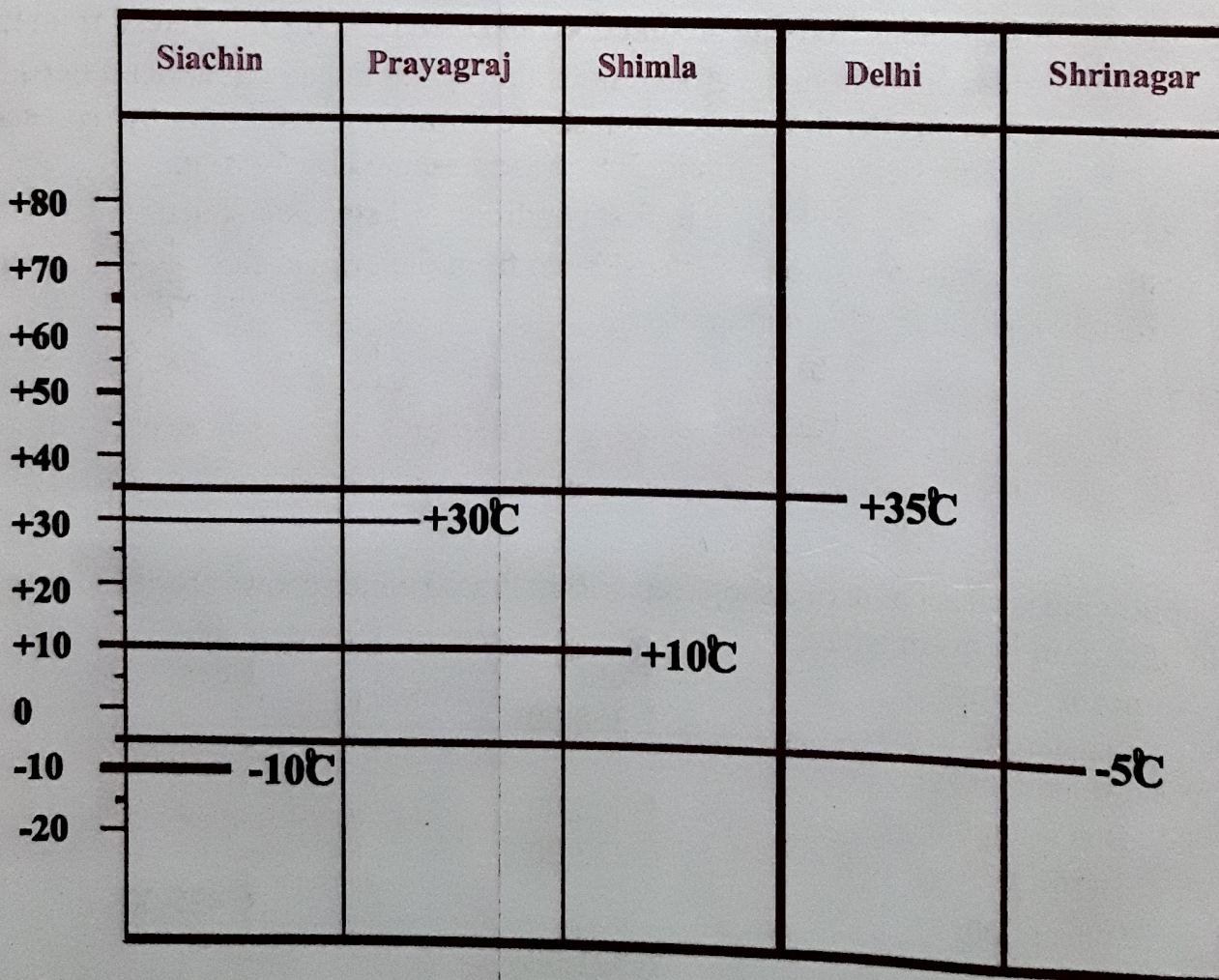
Pay Attention

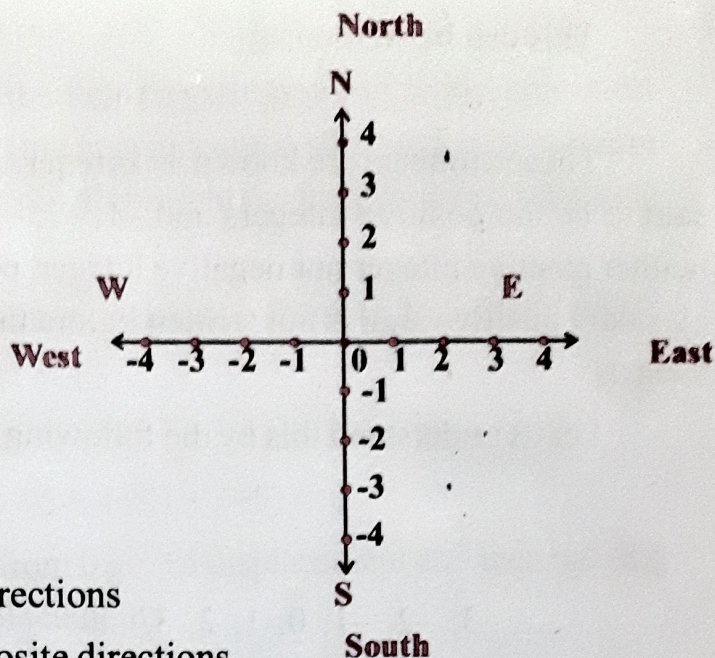
Following is the list of temperatures of five places in India on a particular day of the year.

Place	Temperature
Siachin	10°C below 0°C
Prayagraj	30°C above 0°C
Shimla	10°C above 0°C
Delhi	35°C above 0°C
Sri Nagar	5°C below 0°C



It is evident from the above table that temperature above 0°C is denoted by '+' sign and temperature below 0°C is denoted by '-' sign.





See the adjacent Figure :-

East and West are mutually opposite directions

North and South are also mutually opposite directions.

4 unit West is opposite to 4 unit East.

What is the opposite of the following ?

- (a) 3 unit East (b) 8 unit West (c) 4 unit East
 (d) 2 unit South (e) 7 unit North (f) 9 unit South

- (i) Marking to the East of 0 represent (+) positive and to the West of 0 represent negative (-) find the positions of (a), (b) and (c) of the above question with suitable sign.
 (ii) Marking to the North of 0 represents positive (+) and to the south of 0 represent negative (-). Find the position of above parts (d), (e) and (f) with sign.

We include some new numbers in this collection to extend the collection of whole numbers as needed. Corresponding to the natural numbers 1, 2, 3, 4, we make these new number in the following way.

1 corresponds to -1 (minus one) this makes : $1 + (-1) = 0$

1 and -1 are mutually opposite.

2 corresponds to -2 (minus two) this makes : $2 + (-2) = 0$

2 and -2 are mutually opposite.

3 corresponds to -3 (minus three) this makes : $3 + (-3) = 0$

3 and -3 are mutually opposite.

These new numbers -1, -2, -3 are called negative integers.

If we put the natural numbers, negative numbers and zero together, the collection will be represented as 0, 1, -1, 2, -2, 3, -3.....

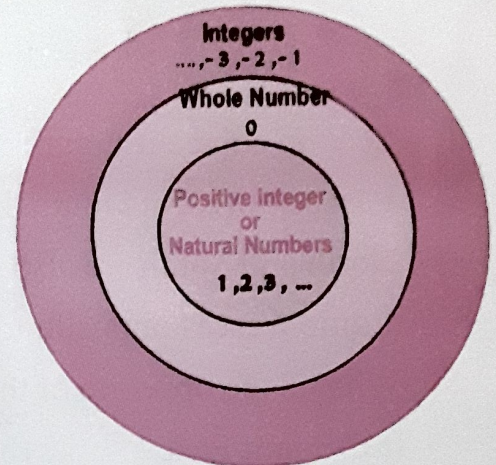
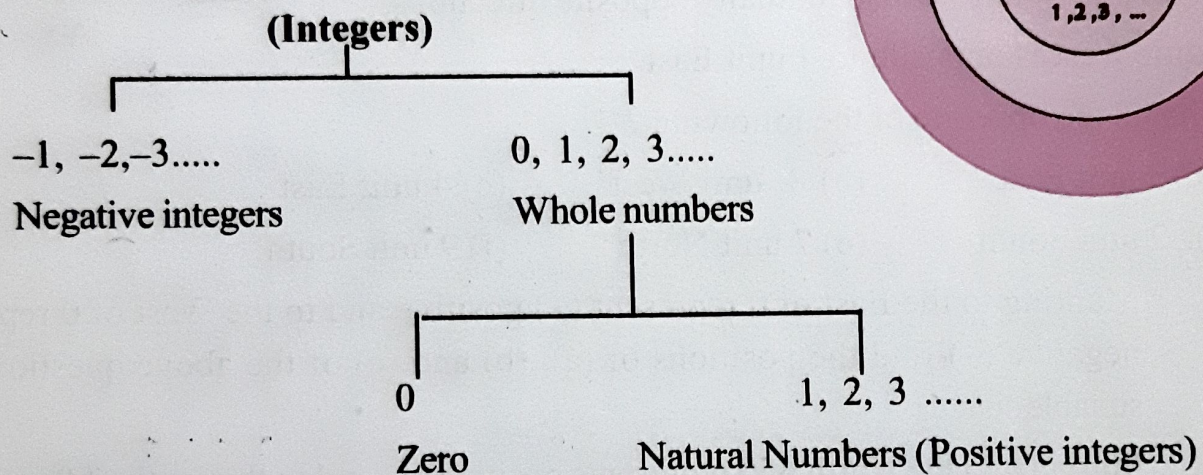
This can be written as

... -3, -2, -1, 0, 1, 2, 3 ...

These numbers are known as **Integers**. In this collection natural numbers 1, 2, 3.... are said to be the positive integers and -1, -2, -3 are said to be negative integers. Zero is neither positive integer nor negative integer, positive integers can be written as +1, +2, +3 Usually positive sign is not written before them. Therefore numbers 1, 2, 3 are positive integers.

Let us understand this by the following figure.

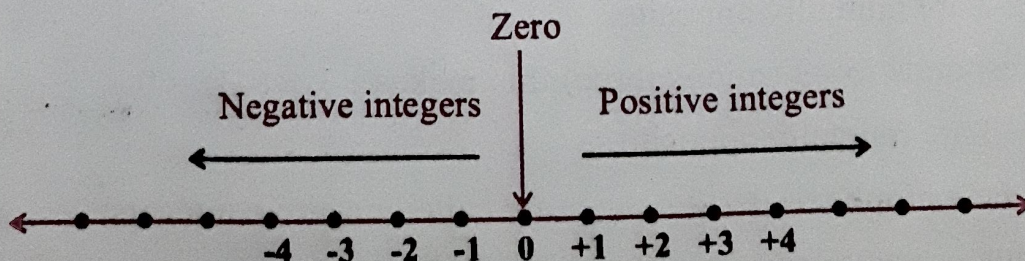
..... -3, -2, -1, 0, 1, 2, 3, ...



3.3 Representation of Integers on Number Line

- Draw a straight line

Take a point 0 in the mid of it and mark some points at equal distance on it as shown in the figure. Mark points to the right of zero as positive integers like +1, +2, +3,..... Points to the left of zero as negative integers like -1, -2, -3



Coincidence of Opposite Numbers

- Draw a line in the playground with the help of rope and line in the east-west direction. Mark a lot of points from line at a successive distance of one metre each of rope from one point in the middle. Represent the integers by representing the point at in the middle as zero.

Go from 0 to 5 metre east to +5. Where do we reach from +5 to 5 metre west?

Displacement from 0 to 5 metre east = +5

5 metre west side displacement = -5

Therefore displacement from +5 to -5 = $(+5) + (-5) = 0$

Similarly, going 5 metre east toward from 0 get +5 displacement and then moving 5 metre west from there get -5 displacement.

What is the resultant of both the displacement ?

$$(+5) + (-5) = 0$$

In this way we can see that

+5 and -5 are such type of opposite integers that $(+5) + (-5) = 0$

Similarly +7 and -7 are mutually opposite integers.

Try these :

1. Find the inverse integer of the following integers :

-6, +9, -11, +12, -17.

2. Evaluate :

$(-11) + (+11)$, $(+9) + (-9)$, $(+13) + (-13)$

Conclusion :

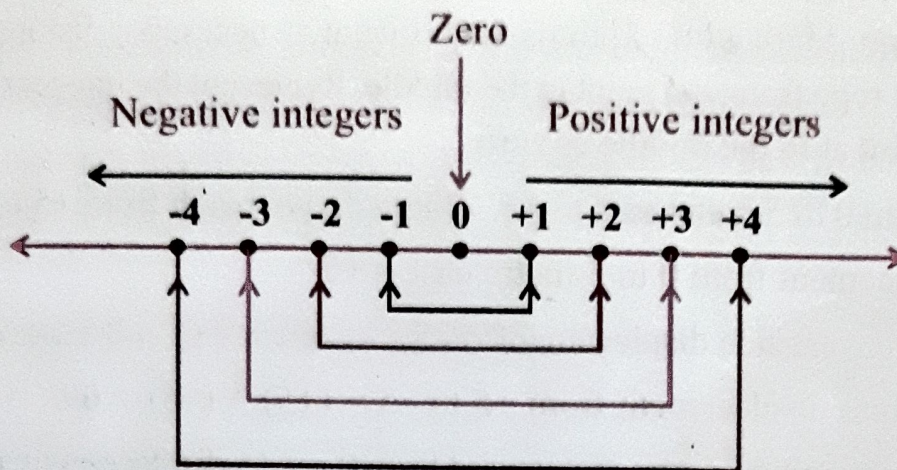
There exists a negative integers corresponding to every positive integer and sum of these two integers is 'zero'.

We see that on a number line.

(i) Positive integers are right to zero.

(ii) Negative integers are left to zero.

Pair of opposite integers on number line



Know the facts

There exists a negative integer corresponding to every positive integer of which distance from zero is same as that of positive integer but in opposite direction.

We can make pair of positive and negative integers that are mutually opposite. Thus we can use these pairs for measurement in opposite direction. e.g. if -40 rupees represents loss then $+40$ rupees represents profits.

....., $-3, -2, -1, 0, 1, 2, 3, \dots$ are integers

$+1, +2, +3, \dots$ are positive integers

$-1, -2, -3, \dots$ are negative integers.

Including zero all positive and negative integers are integers.

Additive Inverse :

If sum of two integers a and b is zero then a and b are additive inverse of each other since, $(+a) + (-a) = 0$

We say that (a) and $(-a)$ are additive inverses of each other.

Since, $(+6) + (-6) = 0$

Therefore, $+6$ and -6 are additive inverse of each other.

Know this :

Since $0 + 0 = 0$

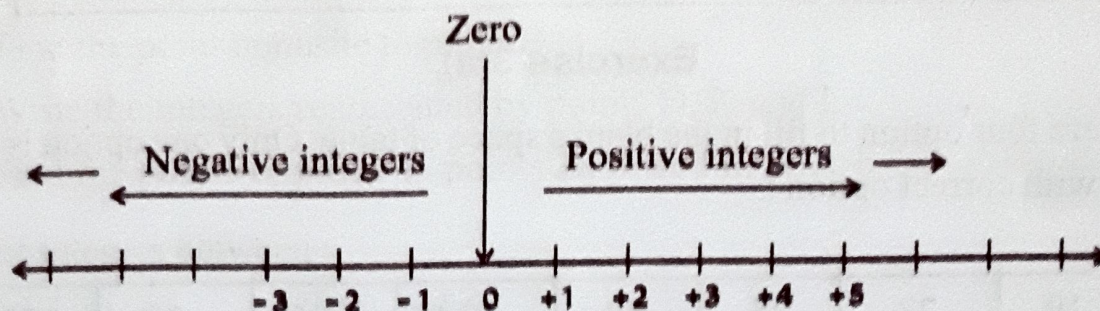
Therefore, zero is additive inverse of itself.

Comparison of Integers :

Think on following :

Which temperature is greater, 5°C above zero or 5°C below zero.

Which number is greater $+5$ or -5



It can be observed in case of whole numbers that the number which lies to the right of a number on the number line is greater than the number which lies to the left side. Since, $8 > 6$ and $6 < 8$ therefore 8 lies to the right of 6.

Same rule is applicable in case of comparison of integers. Thus the integer which lies on the right side of integers is always greater than that on the left.

Remember this :

1. Every positive integer is larger than every negative integer
2. Zero is less than every positive integer.
3. Zero is larger than every negative integer.
4. Additive inverse of larger integer is less than additive inverse of smaller integer e.g. $5 > 3 \Rightarrow -5 < -3$

3.4 Numerical (Absolute) Value of an Integer

Draw a number line and represent integers on it. See the number line and tell

- How much distance is $+6$ from zero ?
- How much distance is -6 from zero ?

What are the relations between these two distances ?

Magnitude of both the distances is 6. We can thus say that numerical value or absolute value of $+6$ and -6 is 6 regardless of its sign. We can write absolute value of -6 as $|-6|$ and absolute value of $+6$ as $|+6|$. In this way

$$\begin{array}{lcl} |-6| = 6 = |+6| & & |-8| = 8 = |+8| \\ |-9| = 9 = |+9| & & |0| = 0 \end{array}$$

Try these :

Fill in the blanks :			
$ +11 =$		$ -13 =$	
$ -47 =$		$ 12 =$	

Exercise 3(a)

There are four options to fill in the blank space of table. Only one option is correct fill in the blanks with correct option :

1.

19	23	25	29	-29	-25	-23	-19
7	11	13	17	-17		-11	-7

(i) -25

(ii) -13

(iii) 13

(iv) -10

2.

-7	25	7	-25	-13	31	13	-31
5	-8	-5	8	11	-14	11	

(i) -13

(ii) 17

(iii) 14

(iv) -17

3. Write the following in form of integers. Take East as Positive direction and the height of a place above sea level as positive number.

(i) 4°C below 0°C temperature

(ii) 7 steps toward east

(iii) loss of ₹ 3.28.

(iv) displacement of 18 metre towards west.

(v) 1000 metre above sea level

4. Write the additive inverse of the following integers :

(i) +9

(ii) -21

(iii) +39

(iv) -41

(v) +91

5. Draw the number line and represent the following integers on it.

(i) +4

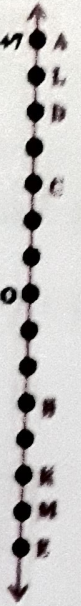
(ii) -9

(iii) +7

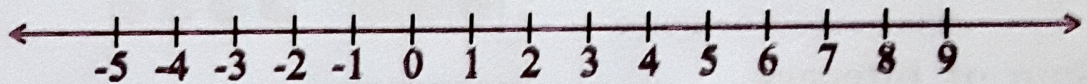
(iv) -3

(v) -6

6. We can represent integers on vertical line on which upward direction is positive and downward direction is negative. See the adjacent figure and answer the following questions :



- (a) Point A is +7, which point is -7 ?
 - (b) Which integer represents point B ?
 - (c) Find the point opposite to point B.
 - (d) Write the integers represented by points D, K and L
 - (e) Find the points opposite to points D, K and L.
7. Find the integers between :
- (i) -2 and +3
 - (ii) 0 and 6
 - (iii) -4 and 4
 - (iv) -5 and +2
 - (v) -3 and +5
 - (vi) -8 and 4
8. See the number line given below and write the integers between given pairs of integers in ascending order.



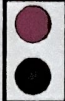
- (i) 2, 5
 - (ii) -4, 0
 - (iii) 3, 8
 - (iv) 0, 4
 - (v) -5, -2
 - (vi) -1, 4
9. Find the values :
- (i) $|-13|$
 - (ii) $|12|$
 - (iii) $|-37|$
 - (iv) $|0|$
 - (v) $|-47|$
 - (vi) $|101|$
10. Check whether the following statement is correct or not :
- (i) Smallest integer is zero.
 - (ii) There is no inverse of zero.
 - (iii) $-17 > -5$
 - (iv) A positive integer is greater than its additive inverse.
 - (v) A negative integer is greater than its additive inverse.
 - (vi) Zero is not an integer.
 - (v) Additive inverse of zero is zero.

3.5 Operations on Integers

Take two different coloured disc like red and black. Let us denote one red disc by positive sign and one black disc by negative sign. Thus one red disc denotes +1 and one black disc denotes -1. A pair of one red '●' and one black '●' disc will denote zero i.e.

$[(+1) + (-1) = 0]$. Therefore pair of  is zero.

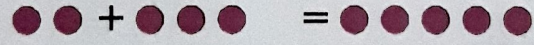
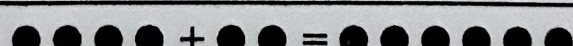
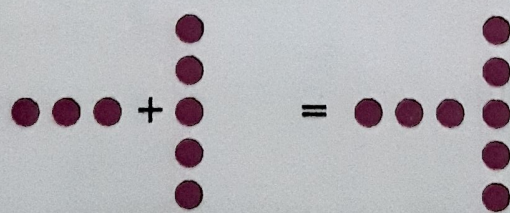
In the following table, integers are shown with the help of coloured discs.

Integers	Representation by coloured discs
4	
-3	
5	
Zero (0)	 or 

3.5.1 Sum of Integers

Addition of integers with the help of coloured discs.

See the table

$(+2) + (+3)$		$(+2) + (+3) = +5$
$(-3) + (-1)$		$(-3) + (-1) = -4$
$(-4) + (-2)$		$(-4) + (-2) = -6$
$(+3) + (+5)$		$(+3) + (+5) = +8$

We see that sum two positive integers is positive and sum of two negative integers is negative.

Try these :

1. Find the values with the help of coloured discs.

(i) $(-4) + (-5)$

(ii) $(+6) + (+4)$

(iii) $(-11) + (-7)$

(iv) $(-2) + (-1)$

(v) $(+4) + (+2)$

(vi) $(-3) + (-3)$

2. Evaluate :

(i) $(-2) + (-1)$

(ii) $(-3) + 0$

(iii) $(+4) + (+5)$

(iv) $(-5) + (-6)$

(v) $(-5) + (-2)$

(vi) $(+6) + (+3)$

(b) Let us perform addition of unlike sign with the help of coloured discs. observe the following :

(i) $(-3) + (+4) =$ Three discs of black colour + four discs of red colour

$$= \begin{array}{c} \bullet \bullet \bullet \\ + \\ \bullet \bullet \bullet \bullet \end{array} = \begin{array}{|c|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline \end{array} \bullet$$

After pairing three black and three red coloured disc, there is one red coloured disc remains.

$$\begin{aligned} (-3) + (+4) &= [(-3) + (+3)] + (+1) \\ &= 0 + (+1) \\ &= (+1) \end{aligned}$$

(ii) $(+5) + (-7) =$ 5 discs of red colour + 7 discs of black colour

$$= \begin{array}{c} \bullet \bullet \bullet \bullet \bullet \\ + \\ \bullet \bullet \bullet \bullet \bullet \bullet \end{array} = \begin{array}{|c|c|c|c|c|} \hline \bullet & \bullet & \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet & \bullet & \bullet \\ \hline \end{array} \bullet \bullet$$

two black coloured discs are remaining.

$$\begin{aligned} &= [(+5) + (-5)] + (-2) \\ &= 0 + (-2) \\ &= (-2) \end{aligned}$$

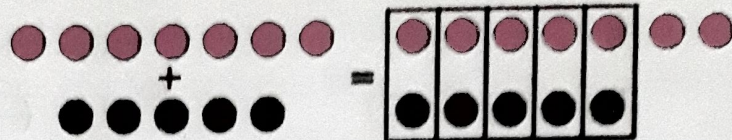
(iii) $(-4) + (+3) =$ four black discs + three red discs

$$= \begin{array}{c} \bullet \bullet \bullet \bullet \\ + \\ \bullet \bullet \bullet \end{array} = \begin{array}{|c|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline \end{array} \bullet$$

$$= \boxed{(-3) + (+3)} + (-1)$$

$$= -1$$

$$(iv) (+7) + (-5) =$$



$$= (+5) + (-5) + (+2)$$

$$= 0 + (+2)$$

$$= (+2)$$

Try these :

● **Find the values :**

(a) $(-12) + (+8)$

(b) $(+7) + (-5)$

(c) $(-12) + (+9)$

(d) $(-27) + (+18)$

(e) $(+15) + (+13)$

(f) $(+31) + (-18)$

(g) $(-19) + (+16)$

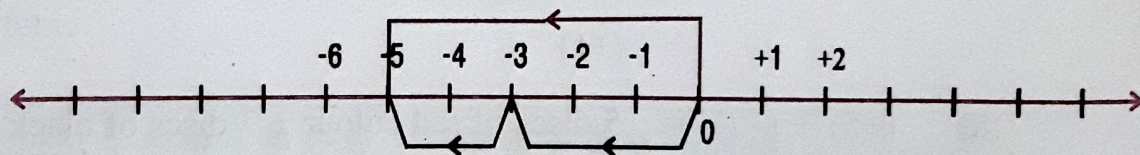
(h) $(-8) + (+12)$

(i) $(-16) + (+19)$

(c) Addition of Integers on a number line :

Example 1 : With the help of number line find the value of $(-3) + (-2)$.

Solution



On the number line, we first move 3 steps to the left of 0 reaching -3 , then we move 2 steps to the left of -3 and reach -5 .

Thus $(-3) + (-2) = -5$

Example 2 : Find the value of $(+6) + (+3)$

Solution

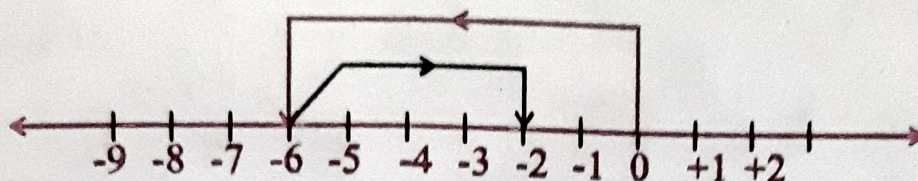
In case of addition of whole numbers we saw that $(+6) + (+3) = +9$

From the above examples we find the rule of addition of integers of same sign.

For the sum of the two integers of the same sign, the sum of the numbers being added, we put the same sign before the absolute values which is the sign of the integers to be added.

(d) Addition of two integers of opposite sign with the help of number line.

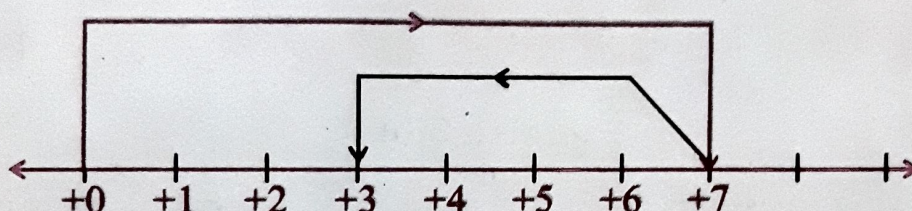
Example 3 : Find the value of $(-6) + (+4)$.



First we move 6 steps to the left of 0 reaching -6 and then from this point we move 4 steps to the right. We reach the point -2 .

Thus $(-6) + (+4) = -2$.

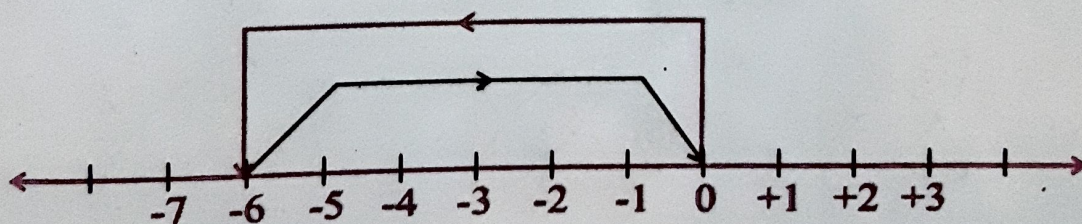
Example 4 : Find the sum of $(+7) + (-4)$



First we move to the right of 0 by 7 steps reaching 7. Then we move 4 steps to the left of 7. where do we reach. we reach 3 unit to the right of 0 i.e. at +3

Thus $(+7) + (-4) = +3$

Example 5 : Find the sum of $(-6) + (+6)$.



First we move 6 steps to the left of 0 reaching -6 and then from this point we move 6 steps to the right where do we reach? We reach the point 0.

$(-6) + (+6) = 0$

Thus we see that –

1. Sum of two positive integers is positive.
2. Sum of two negative integers is negative.
3. When one positive and one negative integer are added, forget the signs and subtract the smaller number from the bigger one. Put the sign of the number having absolute value bigger number in the result so obtained.
(When difference is zero, there will be no sign.)

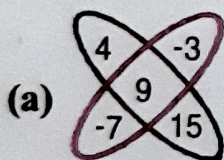
Try These :

Represent the following values on number line :

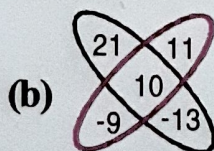
- (a) $(-5) + (+7)$
- (b) $(+5) + (-9)$
- (c) $(-2) + (-6)$

Exercise 3(b)

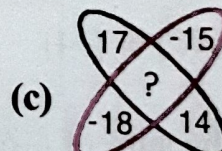
1. In the following questions (1–2) the number of central block is related to any number from the numbers on the sides of the panel. Find the number in place of question marks (?).



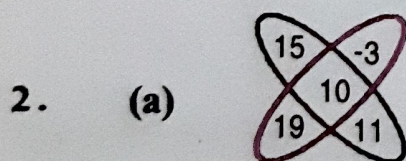
(i) -2



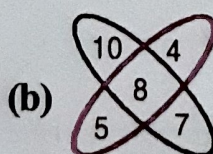
(iii) 3



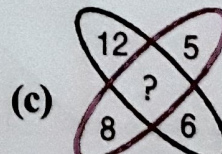
(iv) -3



(i) -5



(ii) 5



(iii) 6

(iv) -6

3. Fill in the blanks :

- (a) Sum of two integers is always
- (b) Sum of two positive integers is
- (c) Sum of two negative integers is

4. State whether true or false.

- (a) Sum of any integers to its additive inverse is always zero.
- (b) Zero is negative integer.

- (c) Sum of two negative integers is positive.
- (d) Sum of any integer and zero is always equal to that integer.
- (e) Sum of negative integer and positive integer is always positive integer.
- (f) On a number line -18 and $+18$ are equidistant from zero.
- (g) Sum of two positive integers is always positive integer.

3.5.2 Properties of Addition Operations on Integers.

1. Fill in the blanks of given table :

Sum of any two integers	Value of Sum	Sum is Integer or not
$(-4) + (+6)$	$+2$	Integer
$(+9) + (-8)$		Integer
$(-12) + (+7)$		
$(+14) + (+3)$		

It is evident from given table that **sum of two integers is always an integer.**

2. Take two integers -7 and $+4$

$$(-7) + (+4) = -3 \text{ and } (+4) + (-7) = -3$$

Similarly take pairs of other integers like -6 and 0 , $+8$ and -11 .

Here $-6 + 0 = -6$ or $0 + (-6) = -6$

and $(+8) + (-11) = -3$ or $(-11) + (+8) = -3$

It is clear from the above that

Sum of two integers in any order remains always same.

Addition is commutative for integers

3. Take any three Integers say $+5$, -3 and -4 .

$$\{(+5) + (-3)\} + (-4) = (+2) + (-4) = -2$$

again $(+5) + \{(-3) + (-4)\} = (+5) + (-7) = -2$

thus, $\{(+5) + (-3)\} + (-4) = (+5) + \{(-3) + (-4)\}$

Similarly check the above property and compare the result for integer (-5) , $(+4)$ and $(+6)$; (-8) , (-3) and $(+9)$

When adding three integers in sequence, begin the addition from the left or start from the right, the sum of the integers are equal.

Addition is associative for integers.

4. See-

$$(-4) + 0 = -4, (+7) + 0 = +7$$

$$(-11) + 0 = -11, 0 + (-5) = -5$$

Thus,

Sum of 0 to any integer is always equal to that integer. We say that 0 is the additive identity for integers.

5. We know that we get the successor of every whole number by adding 1 in it. Every whole number except 0 has its predecessor. Similarly we get the successor of every integer by adding 1 in it. For example $(-5) + 1 = -4$.

Therefore -4 is successor of -5. Since $(-1) + 1 = 0$ hence zero is successor of -1.

Every integer has successor, it gets by adding 1 into that integer.

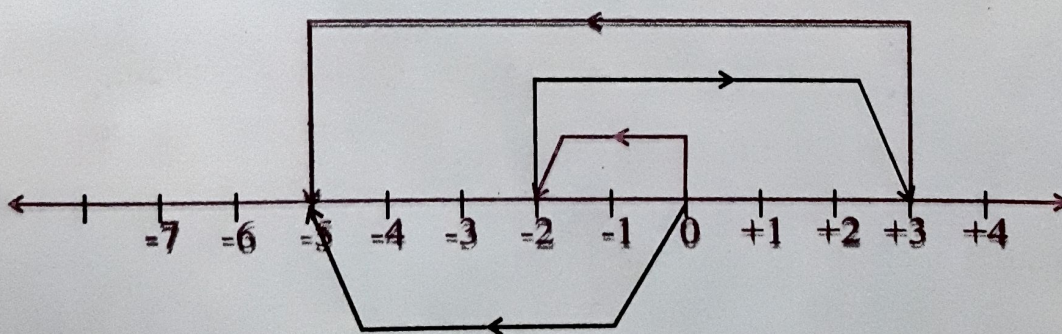
On the basis of this property we conclude that the largest integer does not exist.

Also, there is no smallest Integer. Why?

Example 6

Draw a number line. Represent the addition of $(-2) + (+5) + (-8)$ on it.

Solution



$$(-2) + (+5) + (-8) = -5$$

Try these :

Represent the value of the following on number line.

(a) $(-5) + 7$

(b) $5 + (-9)$

(c) $(-2) + (-6)$

(d) $(-1) + (-2) + 2$

(e) $(-2) + (-3) + (+7)$

3.5.3. Subtraction of Integers

(i) Do this :

Find the values of $(+5) - (+3)$ using coloured discs.

$+5 = 5$ red coloured discs

$+3 = 3$ red coloured discs

$$\therefore (+5) - (+3) = \text{5 red discs} - \text{3 red discs} = +2$$

$$\text{Thus } (+5) - (+3) = +2$$

$$\text{Find the values of } (+5) + (-3) = \boxed{}$$

(ii) Find the value of $(-7) - (-4)$

$-7 = 7$ Black coloured discs

$-4 = 4$ Black coloured discs

$$\therefore (-7) - (-4) = \text{7 black discs} - \text{4 black discs} = -3$$

$$\text{Therefore } (-7) - (-4) = -3$$

$$\text{Find } (-7) + (+4) = \boxed{}$$

(iii) Find $(+3) - (-4)$

$(+3) =$ red coloured 3 discs

$(-4) =$ Black coloured 4 discs

To find the value of $(+3) - (-4)$ withdraw one black coloured disc.

$$\begin{aligned} +3 &= \text{three red coloured discs} \\ &= \text{3 red discs} \\ &= \text{3 red discs} + \text{4 pairs of zeros} \\ &= 3 + (+4 - 4) \\ &= 3 + \text{four pairs of zeros} \\ -4 &= 4 \text{ black discs, drawn} \\ \therefore (+3) - (-4) &= \text{rest of the discs} \\ &= \text{7 red discs} \\ &= +7 \end{aligned}$$

Similarly, $(3) - (-4) = +7$

Here $(3) + (+4) = +7$

(iv) Find the value of $(-2) - (-5)$

$-2 = 2$ discs of black coloured

$-5 = 5$ black coloured discs

$$= \begin{array}{|c|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline \end{array} = -2 + \boxed{(-3) + 3}$$

$(-2) - (-5) =$ we have to draw 2 black coloured discs from 5 black coloured discs.

Similarly,

$$(-2) - (-5) = \begin{array}{|c|c|c|} \hline \bullet & \bullet & \bullet \\ \hline \bullet & \bullet & \bullet \\ \hline \end{array} - \begin{array}{|c|c|c|c|c|} \hline \bullet & \bullet & \bullet & \bullet & \bullet \\ \hline \end{array} = \bullet \bullet \bullet \bullet \bullet$$

Therefore $(-2) - (-5) = +3$

Here $(-2) + (+5) = +3$

Hence, to subtract an integer from another integer it is enough to add the additive inverse of the integer that is being subtracted to the other integer.

Try these :

Find the value :

1. Integers of same sign :

$$(+4) - (+2)$$

$$(-5) - (-3)$$

$$(+6) - (+3)$$

$$(+6) - (+4)$$

$$(+5) - (+4)$$

$$(+3) - (+5)$$

$$(-4) - (-6)$$

$$(+2) - (+3)$$

$$(-3) - (-4)$$

$$(+5) - (+6)$$

2. Integers of opposite sign :

$$(-5) - (+3)$$

$$(+4) - (-2)$$

$$(-3) - (+2)$$

$$(+3) - (-3)$$

$$(-4) - (+3)$$

$$(+5) - (-3)$$

$$(-6) - (+2)$$

Try these :

With the help of above method, find the values of the following :

(a) $(+3) - (+4)$

(b) $(-3) - (-2)$

(c) $(-5) - (+4)$

(d) $(+7) - (+3)$

(e) $(+4) - (+6)$

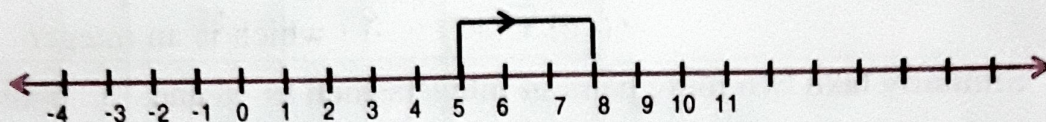
(f) $(-5) - (-3)$

Subtraction of Integers with the help of a Number Line

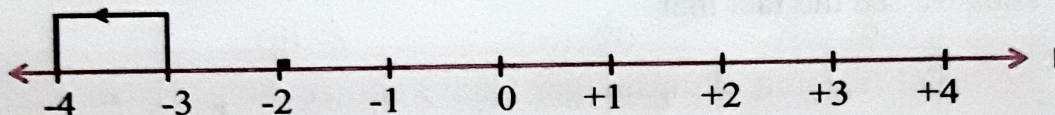
We know that in case of whole numbers $8-5$ means how much and in which direction we have to move from 5 to reach on 8. Similarly in case of integer $(+8) - (+5)$ means, how much and in which direction we have to move so that we should reach $+8$. Since moving 3 steps towards right of $+5$ we reach on $+8$.

Hence $(+8) - (+5) = 3$

this can be represented on number line as :



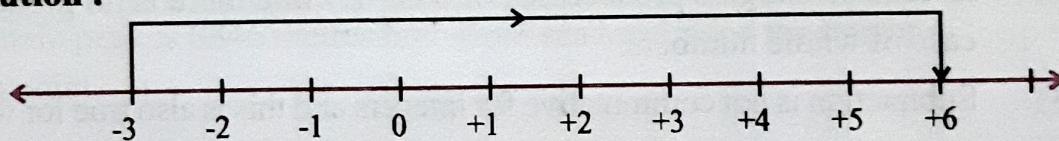
Similarly $(-4) - (-3)$ means that on number line we move in which direction and how much from -3 so that we may reach on -4 .



hence $(-4) - (-3) = -1$

- (i) With the help of number line find the value of $(+6) - (-3)$.

Solution :

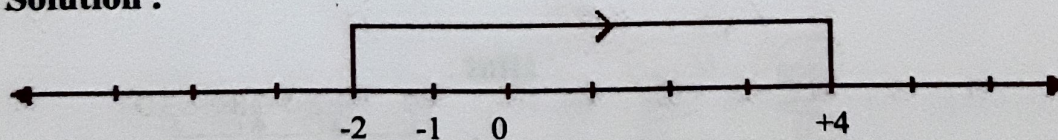


To reach $+6$ from -3 we move towards right by 9 steps.

$$\therefore (+6) - (-3) = +9$$

- (ii) Find the value of $(+4) - (-2)$

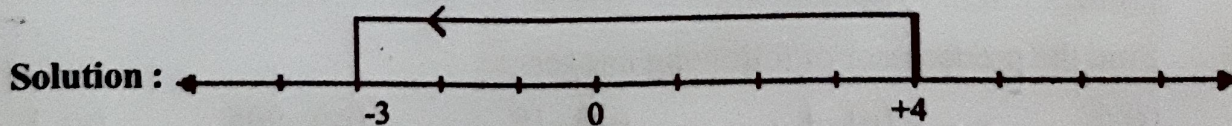
Solution :



to displace from -2 to $+4$ we move 6 steps towards right.

$$\text{hence, } (+4) - (-2) = +6$$

- (iii) Find the value of $(-3) - (+4)$



Since to reach -3 from $+4$ we have to move 7 units towards left.

$$\text{Therefore } (-3) - (+4) = -7.$$

Try these :

With the help of above method find the value of

(a) $(-2) - (-3)$

(b) $(+5) - (+3)$

(c) $(-4) - (-2)$

Properties of Subtraction on Integers :

- (1) Take two integers -6 and 7 .

$$(-6) - 7 = (-6) - (+7)$$

$$= (-6) + (-7) = -13 \text{ which is an integer}$$

Similarly take two more pairs of integers such as -8 and -3 ; 6 and 4 .

$$(-8) - (-3) = (-8) + (+3) = -5 \text{ which is an integer}$$

$$\text{Again } 6 - 4 = 6 - (+4) = 6 + (-4) = +2 \text{ which is also an integer}$$

Thus we see the fact that

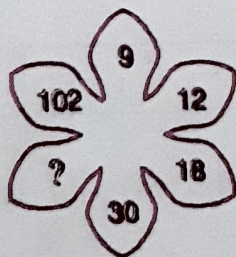
Subtraction of an integer from another integer is an integer.

Therefore integers is closed under subtraction. While whole numbers are not closed for subtraction.

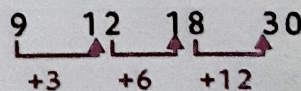
- (2) In case of integers predecessor of 0 is -1 while there is no predecessor of 0 in case of whole numbers.
- (3) Subtraction is not commutative for integers and this is also true for whole numbers.
- (4) Subtracting zero from any integer yields the same integer such as $(-6) - 0 = -6$

Exercise 3(c)

1. Find the number in place of question mark ?



Hint :



(i) 40

(ii) 54

(iii) 45

(iv) 60

2. Find the predecessor of following integers :

(i) 6

(ii) -4

(iii) -19

(iv) -996

(v) 0

3. Fill in the blanks with $>$ or $<$ in your note book.

(a) $(-3) + (-6)$ $(-3) - (-6)$

(b) $(-21) - (-21)$ $(-21) + (-21)$

(c) $(-27) - (+27)$ $27 - (+45)$

4. Fill in the blanks

13	11	9	7
67	?	27	7

(i) 57

(ii) 62

(iii) 47

(iv) 52

5. Find the successor of following integers

(i) 7

(ii) -5

(iii) -20

(iv) -997

(v) 0

6. At a place at 12 noon the temperature was 19°C more than zero and in mid night temperature reduced to 2°C from 0°C . Find the change in temperature.

7. Plato was born in 429 B.C. and died in 348 B.C. How many years he lived ?

8. The maximum depth point from sea level is 11600 metres below. Maximum height of mountain peak is 8846 metres high from sea level. Find the height of the peak from the depth point.

3.5.4. Multiplication Operation on Integers :

We know that multiplication operation is the act of addition repeatedly.

Thus,

$$\begin{aligned} 4 \times (-5) &= (-5) + (-5) + (-5) + (-5) \\ &= -20 \\ &= -(4 \times 5) \\ 3 \times (-6) &= (-6) + (-6) + (-6) \\ &= -18 \\ &= -(3 \times 6) \end{aligned}$$

We now think on $(-3) \times 4$

$$\begin{aligned} (-3) \times 4 &= 4 \times (-3) \text{ (law of commutation of multiplication)} \\ &= (-3) + (-3) + (-3) + (-3) \\ &= -12 \\ &= -(3 \times 4) \end{aligned}$$

$$\begin{array}{rclcl} \text{Similarly} & (-2) \times 5 & = & 5 \times (-2) = -10 \\ & (-8) \times 4 & = & 4 \times (-8) = -32 \\ \text{and} & (-5) \times 7 & = & 7 \times (-5) = -35 \end{array}$$

It is clear from the above that-

The Product of one positive and one negative integer is always negative integer.

Try these :

Fill in the blanks

$$\begin{array}{ll} 5 \times (-4) = \boxed{} & (-3) \times 6 = \boxed{} \\ 7 \times (-3) = \boxed{} & (-4) \times 6 = \boxed{} \end{array}$$

Conclusion :

While multiplying a positive and a negative integer we multiply their absolute value and put a minus sign before the product.

Please Note :

While multiplying two positive integer, we multiply them as a whole number, such as

$$(+7) \times (+4) = +28 = 7 \times 4$$

$$(+8) \times (+3) = +24 = 8 \times 3$$

Product of Two Negative Integers :

If two integers are -3 and -4 , then to know the value of $(-3) \times (-4)$ see the following pattern.

$$\begin{array}{rcl} 4 \times (-4) & = & (-4) + (-4) + (-4) + (-4) = -16 \\ 3 \times (-4) & = & (-4) + (-4) + (-4) = -12 \\ 2 \times (-4) & = & (-4) + (-4) = -8 \\ 1 \times (-4) & = & -4 \\ 0 \times (-4) & = & 0 \\ -1 \times (-4) & = & ? \end{array}$$

Starting from beginning values of product are $-16, -12, -8, -4, 0$ respectively. Thus, product is continuously increasing by $+4$ units.

Thus, according to above rule

$$(-1) \times (-4) = 0 + 4 = 4$$

$$(-2) \times (-4) = 4 + 4 = 8$$

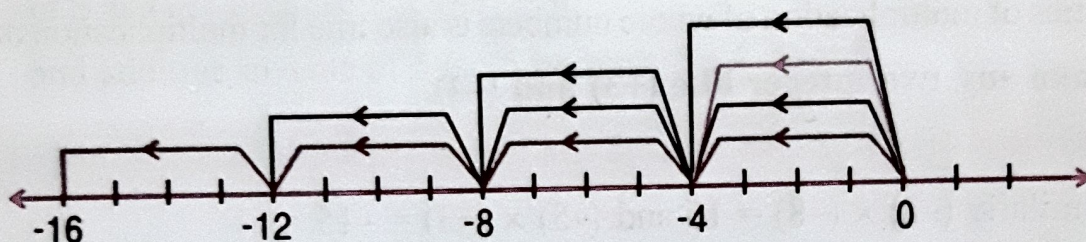
$$(-3) \times (-4) = 8 + 4 = 12$$

$$(-3) \times (-4) = 12$$

Conclusion :

The product of two negative integers is a positive integer.

Multiplication with the help of number line :



$4 \times (-4) =$ displacement of -4 to 4 times.

Hence,

$$4 \times (-4) = -16$$

$3 \times (-4) =$ displacement of -4 to 3 times

Hence,

$$3 \times (-4) = -12$$

$2 \times (-4) =$ displacement of -4 to 2 times.

Hence,

$$2 \times (-4) = -8$$

$1 \times (-4) =$ displacement of -4 to 1 time.

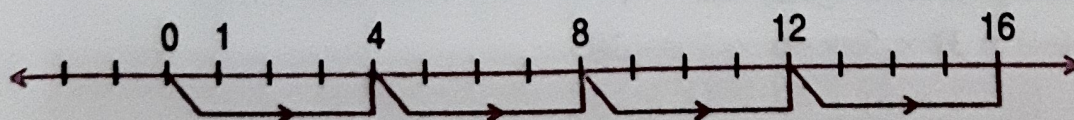
Hence,

$$1 \times (-4) = -4$$

$$0 \times (-4) = 0 \text{ why ?}$$

Therefore, $(-1) \times (-4) =$ displacement of $+4$ to 1 time.

$$= +4$$



$(-2) \times (-4) =$ displacement of $+4$ to 2 times.

$$(-2) \times (-4) = +8$$

Similarly,

$$(-3) \times (-4) = +12$$

$$(-4) \times (-4) = +16$$

Conclusion :

Product of two positive or two negative integers is a positive integer.

Try these :

Fill in the blanks.

$(+3) \times (+6)$	$(-2) \times (-4)$	$(-4) \times (-5)$	$(-0) \times (-1)$
18			

Properties of Multiplication :-

Properties of multiplication of whole numbers is also true for multiplication of integers.

(1) Take any two integer like $(+3)$ and (-4)

$$(+3) \times (-4) = -12$$

$$\text{Similarly } (-2) \times (-8) = 16 \text{ and } (-5) \times (+3) = -15$$

Here the product obtained -12 , 16 -15 are also integers.

Therefore product of two integers is always integers. This is closure property of multiplication of integers.

Please Note :

(2) Since $(+2) \times (-3) = -6$

$$\text{and } (-3) \times (+2) = -6$$

$$\text{hence } (+2) \times (-3) = (-3) \times (+2)$$

Similarly we can check the same process for other pairs of integers.

Product of two integers remains same if we inter change the position of integers. This law is known as Commutative law.

(3) Take any three integers, like

$$(-2 \times 3) \times 5 = -6 \times 5 = -30$$

$$-2 \times (3 \times 5) = -2 \times 15 = -30$$

$$\text{hence } (-2 \times 3) \times 5 = -2 \times (3 \times 5)$$

$$\text{Similarly } (4 \times 5) \times 7 = 4 \times (5 \times 7)$$

$$\text{and } (-6 \times 8) \times 2 = -6 \times (8 \times 2)$$

While multiplying three integers, we start multiplication from left or right, the product remains same.

(4) $2 \times 0 = 0 \times 2 = 0$

$(-3) \times 0 = 0 \times (-3) = 0$

and $0 \times 5 = 5 \times 0 = 0$

The product of any integer with zero is always zero.

(5) $5 \times 1 = 1 \times 5 = 5$; $(-2) \times 1 = 1 \times (-2) = -2$ and $0 \times 1 = 1 \times 0 = 0$

Multiplying any integer by one(1) gives that integer again.

(6) $3 \times (-1) = (-1) \times 3 = -3$; $(-5) \times (-1) = (-1) \times (-5) = 5$

Similarly take any integers and multiply by (-1) and get the result. Additive inverse of 3 is (-3) because $3 + (-3) = 0$

and additive inverse of 5 is -5 because $5 + (-5) = 0$

To know the additive inverse of any integer multiply that integer by (-1) .

Please Note :

(7) (i) $(-2) \times (3 + 5) = (-2) \times 8 = -16$

$(-2) \times 3 + (-2) \times 5 = (-6) + (-10) = -16$

hence $(-2) \times (3 + 5) = (-2) \times 3 + (-2) \times 5$

Take three other integers and do the same process

First Integer $\times$ (Second integer + Third integer)

= First integer $\times$ Second integer + First integer $\times$ Third integer

(ii) Take integer -3 , 7 and 5

$-3 \times (7 + 5) = (-3) \times 12 = -36$

$(-3) \times 7 + (-3) \times 5 = (-21) + (-15) = -36$

hence take three more integers and do the same job.

First integer $\times$ (Second integer - Third integer)

= First integer $\times$ Second integer - First integer $\times$ Third integer

Exercise 3(d)

1. Fill in the blanks :

7	9	11	13
7	27	55	

- (a) 77 (b) 91 (c) 81 (d) 98

2. Evaluate

- (a) $8 \times (-4) \times (-5)$ (b) $(-10) \times (-10) \times (-10)$
(c) $(-2) \times 35 \times (-5)$ (d) $(-8) \times (+57) \times 0$
(e) $(-1) \times (-2) \times (-3) \times (-4)$ (f) $(-4) \times (-8) \times (-12) \times (-5)$

3. Find the value

- (a) $(-7) \times (0) \times (57) \times (-57)$
(b) $1456 \times 625 - 456 \times 625$
Hint : Similarly in the form of $(1456 - 456) \times 625$
(c) $(-187) \times (-54) + (-187) \times (-46)$
Hint : $(-187) [(-54) + (-46)]$
(d) $18764 \times 99 - (-18764)$
(e) $15341 \times (-2) + (-15341) \times 98$
(f) $(-8) \times \{ 10 - 5 - 43 + 98 \}$

4. Product of two integers of Row 'A' are given in Row 'B'.

Match these Products.

- (a) $3 \times (-2)$, $(-4) \times (-5)$, $(-6) \times (4)$, 7×5 , $(-8) \times (-3)$, $5 \times (-3)$
(b) -15, 35, -6, 20, -24, 24

5. Write the integer whose product with (-1) is

- (a) -37 (b) 49 (c) 0

6. Choose the correct answer from the given four options

- (a) $(3 + 2) \times 7 =$
(i) $3 + 2 \times 7$ (ii) $3 \times 7 + 2$
(iii) $3 \times 7 + 2 \times 7$ (iv) $3 \times 7 - 2 \times 7$

(b) Value of $(3-2) \times 6$ is :

(i) $3 \times 6 - 2 \times 6$

(ii) $3 \times 6 - 2$

(iii) $3 \times 6 + 2 \times 6$

(iv) $3 \times 2 \times 6$

(c) Value of $(-8) \times (-5)$ is :

(i) -40

(ii) 40

(iii) 13

(iv)

None of these

(d) While multiplying a positive integer into product of three negative integer, we get the sign of the product as :

(i) Positive

(ii) neither positive nor negative

(iii) Negative

(iv) None of these

7. Find whether the given statement is true or false :

(i) Continuous product of five negative integers is positive.

(ii) Product of two negative integers is positive.

(iii) Product of one positive integer with negative integer is always negative.

(iv) Inverse of -17 is $+17$

(v) To know the inverse of an integer, we multiply by zero in this integer.

(vi) Multiplying by 1 in any integer we get the product as same integer.

3.5.4. Division of Integers :

During the discussion of division of whole numbers, we saw that division is the inverse operation of multiplication

$5 \times 7 = 35$, it means that

$$35 \div 5 = \frac{35}{5} = 7$$

and $35 \div 7 = \frac{35}{7} = 5$

We get two facts of division regarding the product of integer $(-6) \times 3 = -18$

as $-18 \div (-6) = 3$

and $-18 \div 3 = -6$

Observing these examples, we come to the conclusion that if divisor and dividend both are either positive or negative quotient will be positive.

If one of the divisor or dividend is positive and other is negative then quotient will be negative.

✎ Try these :

Find the values of

$$20 \div 4, \quad 28 \div 7, \quad (-15) \div (-3), \quad (-36) \div (-9)$$

It is clear from the above example that

If both divisor and dividend are of same sign (either both are positive or both are negative) quotient will be positive.

Again

✎ Try these :

Evaluate $-16 \div 4, \quad 42 \div (-6), \quad 54 \div (-9), \quad -48 \div 8$

It is clear from the above examples, that

If divisor and dividend both are of opposite sign then quotient will be negative

Example 7 : Find the value of :

(i) $-68 \div 17$

(ii) $78 \div (-13)$

(iii) $(-75) \div (-15)$

(iv) $(+64) \div (+16)$

Solution :

$$(i) \quad -68 \div 17 = \frac{-68}{17} = -4$$

$$(ii) \quad 78 \div (-13) = \frac{78}{-13} = -6$$

$$(iii) \quad -75 \div (-15) = \frac{-75}{-15} = 5$$

$$(iv) \quad 64 \div 16 = \frac{64}{16} = 4$$

Properties of Division :

(i) $6 \div 3 = 2, 10 \div 2 = 5$ and $-8 \div 4 = -2$

here quotients are integers 2, 5 and -2.

but $8 \div 3 = \frac{8}{3}$ (which is not integer)

$$-7 \div 11 = \frac{-7}{11} \text{ not an integer}$$

Therefore set of integers are not closed under division.

(ii) In case of non zero integers

$$3 \div 3 = 1; (-6) \div (-6) = 1 \text{ and } (-2) \div (-2) = 1$$

On dividing a non zero integer by same integer we get quotient 1 always.

(iii) $5 \div 1 = 5$; $-3 \div 1 = -3$ and $0 \div 1 = 0$

If we divide an integer by 1, the quotient is same integer.

(iv) $0 \div 3 = 0$; $0 \div (-2) = 0$ and $0 \div 7 = 0$

If we divide zero by a non zero integer, the quotient is zero.

(v) **Division of any integer by '0' is undefined.**

Exercise 3(e)

1. Find the quotient

(a) $21 \div (-3)$

(b) $(-36) \div 9$

(c) $(-18) \div (-6)$

(d) $35 \div (-7)$

(e) $(-51) \div 17$

(f) $0 \div (-11)$

(g) $(-1728) \div 12$

(h) $(-15625) \div 125$

(i) $(-729) \div (-9)$

(j) $1051 \div (-1)$

(k) $20000 \div (-1000)$

(l) $17672 \div (-17672)$

2. Find the number in place of question mark '?'

(i) $(18 - 3) + (9 \times 2) - 6 = ?$

(a) 12

(b) 9

(c) 27

(d) 39

(ii) $(28 + 4) - (10 \times 5) + (4 \div 2) = ?$

(a) 23

(b) -12

(c) 17

(d) -16

3. Match the following looking at Column A and B.

'A'

'B'

$6 \div 3$

6

$(-12) \div 6$

-4

$18 \div (-6)$

-7

$(-21) \div (-7)$

2

$24 \div 4$

-3

$(-25) \div (-5)$

3

$(-21) \div 3$

-2

$28 \div (-7)$

5

4. Choose the correct answer from the given four option-

(a) Sign of $(-36) \div 9$ is

(i) positive

(ii) negative

(iii) neither positive nor negative

(iv) None of these

(b) Sign of $27 \div (-3)$ is

(i) Neither positive nor negative

(ii) positive

(iii) negative

(iv) None of these

(c) Sign of $(-36) \div (-4)$ is

(i) positive

(ii) neither positive nor negative

(iii) negative

(iv) None of these

(d) Sign of value of $28 \div 7$ is :

(i) negative

(ii) positive

(iii) Neither negative nor positive

(iv) None of these

3.6 Uses of Brackets :

Savita and Merry are two friends. One day they went to market to buy apples. Savita buys 3 kilograms and Merry buys 5 kilograms of apples. If the shopkeeper is selling at ₹40 per kilogram, how many rupees will both together give to the shopkeeper ?

Savita calculated his amount as :

$$= 3 + 5 \times 40$$

$$= 3 + 200$$

$$= ₹ 203$$

Merry first calculate total amount of apple bought = $3\text{kg} + 5\text{kg} = 8\text{kg}$ and then

$$\text{total amount} = 8 \times 40$$

$$= ₹ 320$$

Think

Whose calculation is correct and why ? The shopkeeper says that Merry has rightly calculated and she needs a total of ₹ 320. Both pay ₹ 320 to the shopkeeper, but Savita thought why her calculation is wrong. When Savita comes home and asks her elder sister Suman, Suman explains Savita to calculate using the bracket in such a situation.

$$\begin{aligned}
 \text{Total amount} &= ₹ (3 + 5) \times 40 \\
 &= ₹ 8 \times 40 \\
 &= ₹ 320
 \end{aligned}$$

By proper use of brackets, first obtain the number inside the brackets as a single number and perform the intended calculation on the operation given outside the brackets.

Note and Remember

- (i) In simplifying a given expression, we perform Division, Multiplication, Addition and Subtraction strictly in this order.

$$(a) 35 \div 7 + 4 = 5 + 4 = 9$$

$$(b) 13 - 4 \div 2 = 13 - 2 = 11$$

$$\begin{aligned}
 (c) 55 - 6 \times 3 + 32 - 3 &= 55 - 18 + 32 - 3 \\
 &= 55 + 32 - 18 - 3 \\
 &= 87 - 21 \\
 &= 66
 \end{aligned}$$

- (ii) If we say divide 36 by the product of 4 and 3, then we can write as :

$$36 \div (4 \times 3)$$

If we write this as $36 \div 4 \times 3$

then answer is $9 \times 3 = 27$ which is incorrect.

Similarly $48 \div (3 + 5)$ means. First add 3 and 5 and then divide 48 by this sum.

If we don't use bracket () then its form is $48 \div 3 + 5$, Which means

$$48 \div 3 + 5 = 16 + 5 = 21$$

$$\text{while } 48 \div (3 + 5) = 48 \div 8 = 6$$

hence we use brackets to avoid this type of mistakes.

- (iii) Sometimes the following types of problems are encountered.

Find the quotient when 120 is divided by an integer 8 larger than the product of 4 and 3.

Take the product of 4 and 3 as (4×3)

8 large than product of 4 and 3 can be written as $|(4 \times 3) + 8\}$

Thus, required quotient is $= 120 \div \{(4 \times 3) + 8\} = 6$

In doing so a second type of bracket $\{3\}$ is used in mathematics.

The following four types of brackets (paranthesis) are generally used.

Symbol	Name
—	vinculum
()	paranthesis; round bracket
{ }	braces, curly bracket
[]	square bracket

The left portion of each bracket represents the beginning of the bracket, and the right portion of the bracket represents end.

- (i) Simplification is done by removing the Vinculum, round bracket, curly bracket and square bracket. The rule is that the brackets, whichever is in order, are first removed by the order of the inner brackets.
- (ii) If the plus (+) sign is before the bracket, the plus (+) and (–) minus marks remain intact inside the bracket when the bracket is removed.
Similarly if there is (–) minus sign before the bracket, the plus (+) and minus (–) marks inter changes inside the bracket when the bracket is removed.
- (iii) If there is a number before or after the bracket, and there is no sign between that number and the bracket, we should understand the sign of multiplication.
- (iv) Always simplify inside the bracket before removing the bracket.

Example 8 : Evaluate

$$18 - [4 + \{16 - (18 - 5)\}]$$

Solution :

$$\begin{aligned} 18 - [4 + \{16 - (18 - 5)\}] \\ &= 18 - [4 + \{16 - 13\}] \\ &= 18 - [4 + 3] \\ &= 18 - 7 = 11 \end{aligned}$$

BODMAS Rule :

If there is use of 'of', $\div$, $\times$, $+$, $-$, () in any expression the symbols expressed by the BODMAS letters are simplified sequentially.

B = Bracket ()

O = Of

D = Division $\div$

M = Multiplication $\times$

A = Addition $+$

S = Subtraction $-$

This, Simplification of Brackets is convenient by 'BODMAS' rule.

Example 9 Simplify $14 - [12 - \{9 - (7 - \overline{6 - 2})\}]$

Solution : $14 - [12 - \{9 - (7 - \overline{6 - 2})\}]$
 $= 14 - [12 - \{9 - (7 - 4)\}]$ removing vinculum
 $= 14 - [12 - \{9 - 3\}]$ removing round bracket
 $= 14 - [12 - 6]$ removing curly bracket
 $= 14 - 6$ removing square bracket
 $= 8$

Example 10: Simplify $(-12) + (-6) \div 2 - [(-5) \times (-4) - \{2 - (3 - 5)\}]$

Solution :
 $(-12) + (-6) \div 2 - [(-5) \times (-4) - \{2 - (3 - 5)\}]$
 $= (-12) + (-6) \div 2 - [(-5) \times (-4) - \{2 + 2\}]$
 $= (-12) + (-6) \div 2 - [(-5) \times (-4) - 4]$
 $= (-12) + (-6) \div 2 - [20 - 4]$
 $= (-12) + (-6) \div 2 - 16$
 $= -12 + (-3) - 16$
 $= -15 - 16 = -31$

Exercise 3(f)

1. Evaluate the following :

(i) $21 + 18 \div 3$

(ii) $123 - 81 \div 9$

(iii) $13 - (8 \times 2) + 3$

(iv) $12 - (13 - 12 \div 3)$

(v) $28 - 5 \times 7 + 7$

(vi) $117 \div (7 + 6)$

(vii) $(-17) + 8 \div (7 - 3)$

(viii) $(-3) + (-6) \div (-3)$

(ix) $17 + (-2) \times (-5) - 4$

(x) $13 \div \overline{4-3}$

(xi) $(-36) \times (-1) + (-24) \div 6$

(xii) $(-5) - (-45) \div (-15) + (-3) \times 5$

2. With the help of brackets, write a mathematical expression for the following statements.

- (a) Multiplication of 8 with the sum of 6 and 3.
- (b) In 18, divide by the sum of 4 and 2.
- (c) In 20, divide by the difference of 6 and 2
- (d) Subtract 12 from the product of 4 and 5.
- (e) In 40 divide by 1 more the sum of 5 and 2.
- (f) In 3 multiply by 1 less the difference of 7 and 12.

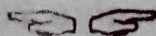
3. Simplify:

- (a) $20 + \{9 - 5 + (6 - 4)\}$
- (b) $80 \times [56 - \{7 \times 8 + (13 - 2 \times 5)\}]$
- (c) $121 \div [16 - \{14 - 3(9 - 6)\}]$
- (d) $5 [18 + \{3 + 6(5 - 3)\}]$
- (e) $16 + \{1 + (16 - 3) \times 4\}$
- (f) $3 - [3 - \{3 - (3 - \overline{3-3})\}]$
- (g) $112 - [121 \div (11 \times 11) - (-4) - \{3 - \overline{8-1}\}]$
- (h) $(-2) \{(-5) + (-25)\} \times (-7) - (4 - 6) (-5)$
- (i) $4 \text{ of } [25 - 18 \div \{7 - 2 \text{ of } 3 - (13 - \overline{4-3}) + 5\}]$

What have we discussed ?

1. In our daily life we have seen that there are times when we need to use numbers with a negative sign. In order to fulfill this need we extend the set of whole numbers. Thus we make number $-1, -2, -3 \dots$ corresponding to Natural numbers $1, 2, 3 \dots$

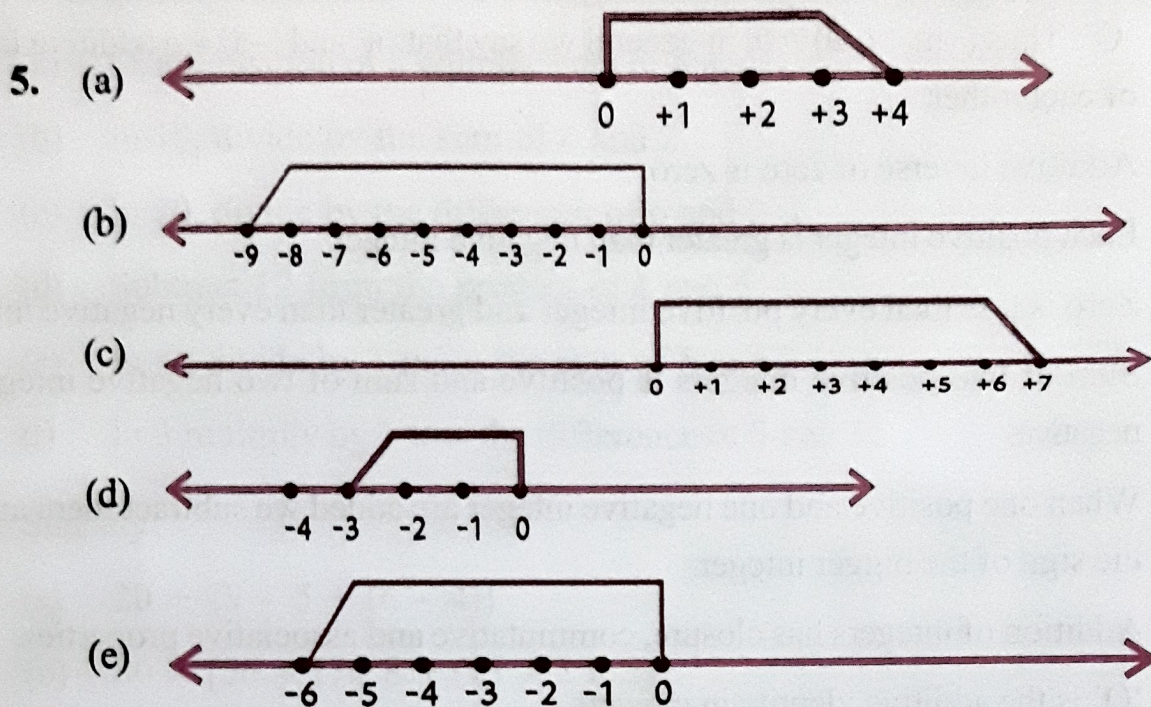
2. In the collection of integers 1, 2, 3 ... are called natural numbers or positive integers whereas $-1, -2, -3 \dots$ are called negative integers. Zero is neither a positive nor a negative integer.
3. Integers can be represented on a number line. There exist a negative integer ' $-a$ ' corresponding to positive integer ' $+a$ '. Sum of both the integers is always ' 0 '. Thus $(+a) + (-a) = 0$, in general we say that ' a ' and ' $-a$ ' are additive inverse of each other.
4. Additive inverse of zero is zero.
5. Each positive integer is greater than negative integer.
6. Zero is less than every positive integer and greater than every negative integer.
7. Sum of two positive integers is positive and sum of two negative integers is negative.
8. When one positive and one negative integer are added we subtract them and put the sign of the bigger integer.
9. Addition of integers has closure, commutative and associative properties.
10. ' 0 ' is the additive identity in integers.
11. Integers are closed (closure property) under subtraction but subtraction is neither commutative nor associative.
12. The product of two integers with same signs is a positive integer but the product of two integers with opposite signs is a negative integer.
13. Multiplication of integers holds closure property, commutative law and Associative law.
14. Division of integer does not hold closure property, commutative law or Associative law.
15. We use brackets to avoid mistakes in calculations.
16. Simplification is done by removing the vinculum, round bracket, curly bracket and square brackets. The rule is that the brackets, whichever is in order, are first replaced by the order of the inner brackets.



ANSWERS

Exercise 3(a)

1. (ii), 2. (iii), 3. (i) -4°C (ii) $+7$, (iii) $-\text{₹ } 3.28$, (iv) -18 , (v) $+1,000 \text{ m}$, 4. (i) -9 , (ii) $+21$, (iii) -39 , (iv) $+41$, (v) -91



6. (a) E, (b) -3 , (c) C, (d) $+5, -5, +6$, (e) K, D, M; 7. (a) $-1, 0, +1, +2$, (b) 1, 2, 3, 4, 5; (c) $-3, -2, -1, 0, 1, 2, 3$; (d) $-4, -3, -2, -1, 0, 1$; (e) $-2, -1, 0, 1, 2, 3, 4$; (f) $-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3$; 8. (a) 3, 4, (b) $-3, -2, -1$; (c) 4, 5, 6, 7; (d) 1, 2, 3; (e) $-4, -3$; (f) 0, 1, 2, 3; 9. (a) 13, (b) 12, (c) 37, (d) 0, (e) 47, (f) 101; 10. (a) False, (b) True, (c) False, (d) True, (e) True, (f) False, (g) True.

Exercise 3(b)

1. (i) 2. (ii) 3. (a) Integer, (b) Positive Integer, (c) Negative integer, 4. (a) True, (b) False, (c) False, (d) True, (e) False, (f) True, (g) True.

Exercise 3(c)

1. (ii) 2. (i) 5, (ii) -5 , (iii) -20 , (iv) -997 , (v) -1 ; 3. (a) $<$, (b) $>$, (c) $<$; 4. (iii) 5. (i) 8, (ii) -4 , (iii) -19 , (iv) -996 , (v) 1; 6. -21°C ; 7. 81 year. 8. 20446 m.

Exercise 3(d)

1. (b) 2. (a) 160, (b) -1000 , (c) 350, (d) 0, (e) 24, (f) 1920; 3. (a) 0, (b) 625000, (c) 18700, (d) 1876400, (e) -1534100 , (f) -480 ; 4. $3 \times (-2) \rightarrow -6$, $(-4) \times (-5) \rightarrow 20$, $(-)$

$6) \times 4 \rightarrow -24$, $7 \times 5 \rightarrow 35$, $(-8) \times (-3) \rightarrow 24$, $5 \times (-3) \rightarrow -15$; 5. (a) 37, (b) -49, (c) 0; 6. (a) (iii), (b) (i), (c) (ii), (d) (iii); 7. (i) False, (ii) True, (iii) True, (iv) True, (v) False, (vi) True

Exercise 3(e)

1. (a) -7, (b) -4, (c) 3, (d) -5, (e) -3, (f) 0, (g) -144, (h) -125, (i) 81, (j) -1051, (k) -20, (l) -1; 2. (i) (c) 27, (ii) (d) -16, 3. $6 \div 3 \rightarrow 2$, $-12 \div 6 \rightarrow -2$, $18 \div -6 \rightarrow -3$, $-21 \div -7 \rightarrow 3$, $24 \div 4 \rightarrow 6$, $-25 \div -5 \rightarrow 5$, $-21 \div 3 \rightarrow -7$, $28 \div -7 \rightarrow -4$; 4. (a) (ii) (b) (iii) (c) (i) (d) (ii).

Exercise 3(f)

1. (i) 27, (ii) 114, (iii) 0, (iv) 3, (v) 0, (vi) 9, (vii) -15, (viii) -1, (ix) 23, (x) 13, (xi) 32, (xii) -23; 2. (a) $(6 + 3) \times 8$, (b) $18 \div (4 + 2)$, (c) $20 \div (6 - 2)$, (d) $(4 \times 5) - 12$ (e) $40 \div \{(5 + 2) + 1\}$, (f) $3 \times \{(12 - 7) - 1\}$; 3. (a) 26, (b) -240, (c) 11, (d) 165, (e) 69, (f) 0, (g) 103, (h) -430, (i) 112.

