

UNIT-5 ALGEBRAIC CONCEPTS



- Requirement and use of letters to denote the numbers.
- The basic operations of numbers and letters.
- Power of letter numbers.
- Variable and Constant numbers.

5.1 Introduction

We have studied numbers and various operations on them in arithmetic. We have used the numbers to solve most of our daily life problems. Many times, complicated problems related to numbers are not solved by arithmetic methods. In addition, the problems need to be resolved more effectively, which we do in another way. Now, we will study the branch of mathematics related to that, which is called Algebra.

In our country, algebra has been studied since ancient times. Algebra is actually a broad form of arithmetic. It's main feature is that it uses letters instead of numbers. **Using letters, we will be able to write rules and formulas in a comprehensive way. The other thing is that characters can be used even in the place of unknown quantities. By writing the methods of determining there unknown odds, we can develop many effective means of solving many problems related to puzzles and daily life, using the letter numbers. We can only use a special number.** You can talk about any number without talking about sepcial number, that is why they are called algebra. With the help of algebra, the rules and formulas can be widely written. Operations like multiplication, division etc. can be performed on the number of letters obtained using letters for numbers too. You will find that algebra is not only useful, but it is also very interesting.

5.2 Use of letters to denote the numbers, read it and try to understand.

- How many apples are in the basket of Figure-1 ?

There are 2 apples in the basket.



Fig. 1

- How many apples are in the basket of Figure-2 ?

There are 3 apples in the basket.



Fig. 2

- How many apples are in the basket of **Figure-3** ?

There are 5 apples in the basket.

In the basket of **Figure 4**, apples are full, tell the correct number.

It is difficult to count the fruits. So only we can say that there are 'some' apples in the basket.

Squares are made in **Figure 5**. See them and tell the length of the side of each square.



Fig. 3



Fig. 4

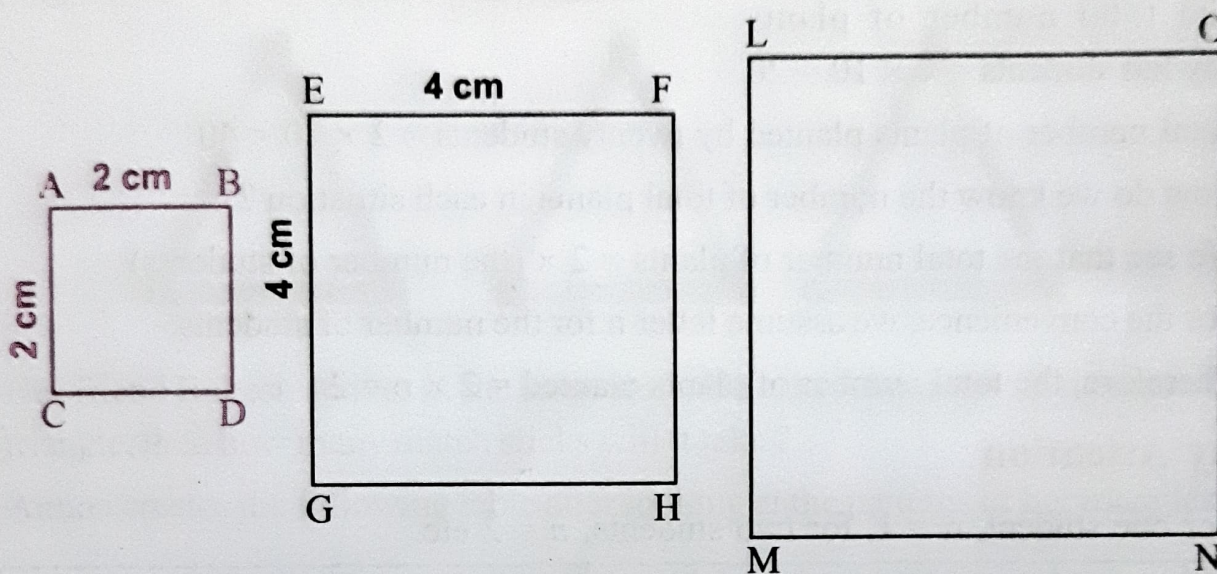


Figure 5

The length of the arm of the square $ABDC = 2$ cm.

The length of the arm of the square $EFHG = 4$ cm.

The length of the side of the square $LMNO$ is not known. We might consider it a 'few' cm.

- Ram had 45 rupees. He got some money from his sister Sheela. Now, how many rupees he has ? Ram has a total of ₹ $(45 + \text{some})$.

In the above examples, we have seen that the numbers in mathematics so far are denoted by 0, 1, 2, 3 ..., 8, 9. We also have to talk about numbers that are not known but can be known. For such numbers, English alphabets (or Hindi) is used.

Suppose, a tree planting program is being organized in a school. Each student planted 2 plants. The first student plant two plants.

Hence, the number of plants = 2

The second student also plant two plants, So the total number of plants = $2 \times 2 = 4$.

The third student also plant two plants, So the total number of plants = $2 \times 3 = 6$

In the same way

The total number of plants planted by ten students = $2 \times 10 = 20$

Total number of plants planted by twenty students = $2 \times 20 = 40$

How do we know the number of total plants in each situation ?

We see that the total number of plants = $2 \times (\text{the number of students})$

For the convenience, we assume letter n for the number of students.

Therefore, the total number of plants planted = $2 \times n = 2n$



Pay Attention

For one student, $n = 1$, for two students, $n = 2$ etc.

Thus, n can be any natural number 1, 2, 3,

For the number of plants planted by 50 students, we will take, $n = 50$. So, number of plants = $2 \times 50 = 100$. Is it not easy to find the total number of plants by the formula $2n$ expressed by the letter n ?

Number of Students	1	2	3	...	...	...	10	...	...	20	...	50	...	n
Number of Plants	2	4	6	...	...	...	20	...	...	40	...	100	...	$2n$

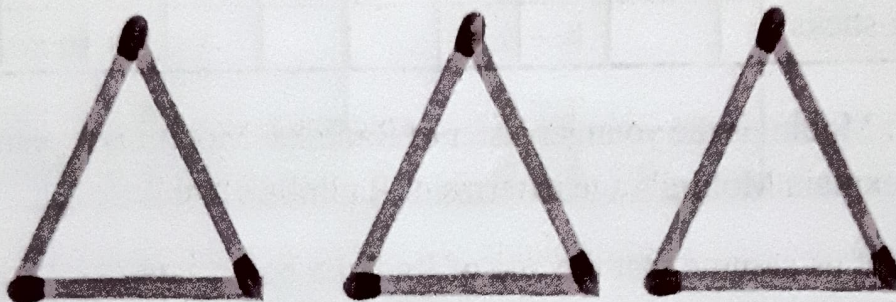
Express any unknown number from English alphabets $x, y, z; a, b, c$ etc.

These numbers are called **letter number**.

In the above mentioned examples, we can replace some 'apples' by ' a ' apple, instead of 'some' cm length of the arm (side) of square, ' y ' cm length and money given by Sheela to Ram, 'some' rupees by ' x ' rupees. The letters used a, y, x are letter numbers. These letter number do not reveal any amount, only express numbers.

Pay Attention to the following statements :

1. Saida has ' n ' mangoes. What is known about the number of mangoes from this statement ? It is known from the statement that Saida has some mangoes whose fixed numbers are not known. These can be any value of n .
2. Shyam had ₹ 4357. He gave some money to Manisha. Let us say that Shyam gave ₹ x to Manisha, so he has total of ₹ $(4357 - x)$ left.
3. Amina is making copies of the equilateral triangles with the match sticks.



Her friend Rakesh arrives at the same time. He asks Amina that if there are 6 copies of this triangle, then how many match sticks will it take ?

Amina creates the following table after looking at the patterns of her triangles :

Number of triangles created	1	2	3	4	5	6
Required number of match-sticks	3	6	9	12	15	18

Amina tells Rakesh that to make 6 copies of equilateral triangles, 18 triangles will be needed.

☞ Try it :

Tell that if the number of copies of the above mentioned triangles is n , then what is the total number of sticks required ?

Example 1 : Find out the rules for the number of essential match sticks needed to form the pattern V (in the form of V). Use a variable to write rules.

Solution :

The required number of match-sticks needed to form V, one copy of V = $2 \times 1 = 2$

The required number of match-sticks needed form VV, two copies of V = $2 \times 2 = 4$

form VVV, three copies of V, number of match-sticks required $= 2 \times 3 = 6$

.....

Hence, to make n copies of V, required number of match-sticks $= 2 \times n = 2n$

We can demonstrate this rule by following table :

Number of copies of V created	1	2	3				10			n
Required number of Match-sticks	2	4	6				20			$2n$

Example 2 : Mohini is the younger sister of Radhika. Mohini is 5 years younger than Radhika. Can you explain Mohini's age in terms of Radhika's age ?

Solution : Let us assume that the age of Radhika be y years.

It is given that Mohini is 5 years younger than Radhika.

Hence, the age of Mohini $= (y - 5)$ years.

Practice 5 (a)

- Teacher give 3 pencils to each student. If there are a total of x students in the class, then tell how many pencils teacher give to the students ?
- Write a term (letter number) instead of numbers in the empty spaces on your copy and tell what number it has been used for :

(i) $12 + 5 = 17$ (ii) $40 - 10 = 30$

$\square + 5 = 17$ $40 - \square = 30$

(iii) $4 \times 6 = 24$ (iv) $35 \div 5 = 7$

$\square \times 6 = 24$ $35 \div \square = 7$

- Rahim had ₹ 10, he gave Razia a few rupees. How many rupees he has left ? Express this relation with the help of letter numbers.
- There are some trees in a garden. The number of trees has increased to 120 after planting 50 trees. Express this statement with the help of letter numbers.
- Find out the rule to know the number of match-sticks required to frame the models of letter N and M. Use a variable to write the rule.

5.3. Basic operations on numbers and letter numbers

Letter numbers $x, y, z, \dots, a, b, c \dots$ etc. are used to show (denote) numbers. All original operations that are used for numerical numbers are also used for letter numbers. You can understand this fact from following tables :

(i) Operation of Addition :

Numbers	Sum
Addition of 5 and 4	$5 + 4$
Addition of x and 4	$x + 4$
Addition of x and y	$x + y$
Addition of a, b and c	$a + b + c$

(ii) Operation of subtraction :

Subtracting 3 from 10	$10 - 3$
Subtracting 5 from x	$x - 5$
Subtracting x from 5	$5 - x$
Subtracting y from x	$x - y$
Subtracting 0 from x	$x - 0$

(iii) Operation of Multiplication :

The multiplication operation is very similar to the operation of addition repeatedly. Look at this carefully :

$2 + 2 + \dots$ 4 times	$= 4 \times 2 = 8$
$x + x + \dots$ 4 times	$= 4 \times x = 4x$
$a + a + \dots$ 3 times	$= 3 \times a = 3a$
$x + x + \dots$ y times	$= y \times x = yx$
$1 + 1 + \dots$ x times	$= x \times 1 = x$
$0 + 0 + \dots$ x times	$= x \times 0 = 0$

(iv) Operation of Division :

Dividing 20 by 5	$20 \div 5 = \frac{20}{5} = 4$
Dividing a by b	$a \div b = \frac{a}{b}, b \neq 0$

Dividing 40 by p $40 \div p = \frac{40}{p}, p \neq 0$

Dividing b by 5 $b \div 5 = \frac{b}{5}$

Pay Attention

- $4 \times x$ is usually written as $4x$. Do not put the multiplication sign in the middle.
- xy means $x \times y$.
- $1 \times x$ is written without writing $1x$ and only x is written.
- $a \div b = \frac{a}{b}, b \neq 0$ and $b \div a = \frac{b}{a}, a \neq 0$

So, $a \div b$ is not equal to $b \div a$, unless $a = b$ or $a = -b$.

We express it as :

$a \div b \neq b \div a$ [$\neq$ is read as 'not equal'] i.e., $\frac{a}{b} \neq \frac{b}{a}$ unless $a = b$ or $a = -b$.

- We can see from the example that the following law of multiplication of numbers also applies to the multiplication of :
 - $xy = yx$ (Commutative law)
 - $(xy)z = x(yz)$ (Associative law)
 - $x(y + z) = xy + xz$ (Distributive law)

Thus, by looking at the examples, we can see that like numbers, letter numbers also follow the operation of addition, subtraction and division process.

Try it :

- Find the sum of the letter numbers x and 7.
- What will be the difference of subtracting y from 5 ?
- Find the sum abc and d .
- Find the value of $o \times y$.
- If $x = 1$ and $y = 1$, then what is the value of xy ?
- What will be the quotient for dividing x by 30 ?

Example 3 : One number is x and second number is 10, what will be the sum of them ?

Solution : Given,

First number = x

Second number = 10.

Then, Sum = $x + 10$

Example 4 : Loss is the difference between cost price and selling price, (when selling price is less than cost price). Express this statement with the help of the signs of the basic operations.

Solution : Let the cost price be ₹ C .

Selling price = ₹ S .

Loss = ₹ L

Since, Loss = Cost price – Selling price

Therefore, ₹ $L = ₹ C - ₹ S$

$\therefore L = ₹ (C - S)$.

Exercise 5(b)

1. Write the following in algebraic form :

(i) Sum of 6 and x .

(ii) Subtracting 7 from x .

(iii) 5 times of x .

(iv) One third of x .

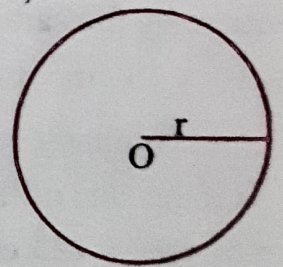
2. Express the following statements with the help of sign of numbers, and basic operations :

(i) The diameter of the circle is twice that of its radius.

(ii) The perimeter of the square is 4 times its one arm length.

(iii) Area of the rectangle is the product of its length and breadth.

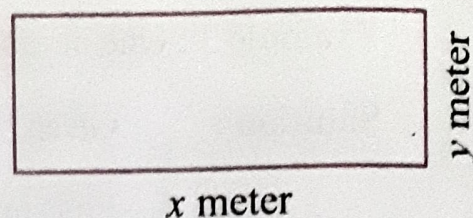
(iv) Profit is the difference between selling price and cost price, when the selling price is more than the cost price.



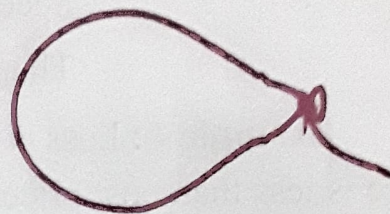
3. (a) In one basket, there are 50 mangoes and in a second basket there are x mangoes. How many mangoes are in the first basket more than in the second basket.

(b) There are 100 students in a school. x students out of them have gotten ill by drinking polluted water. Find out the number of healthy students.

4. The adjacent sides of the rectangle in the attached diagram are x meters and y meters. Write the perimeter of the rectangle.



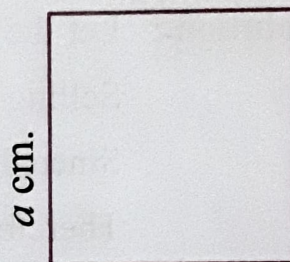
5. The value of a balloon is ₹ x . What is the price of such 12 balloons ?



6. There are x students in the class, of which there are about one fourth are girls. How many girls are in the class ?

a cm.

7. In the attached diagram, the side of a square is ' a ' cm. Write the perimeter of the square.



8. Leela is Radha's younger sister. She is 6 years younger than Radha. Write the age of Leela in terms of Radha's age.
9. Mother made 16 laddoos in the house, She gave x laddoos to guests and family members. Tell how many laddoos are left ?

1.4 Power of letter numbers

You know that in order to find the area of a square, we have to multiply the side of the square by the same side. If the side of the square is 10 cm., then its area = $10 \text{ cm} \times 10 \text{ cm} = 10 \times 10 \text{ cm}^2$. In brief, we write it as $10^2 \text{ cm}^2 = 100 \text{ cm}^2$.

Similarly, to find the volume of a cube, the formula; volume = side $\times$ side $\times$ side is used. As if the side of the cube is 5 cm., then its volume = $5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm} = 5 \times 5 \times 5 \text{ cm}^3 = 5^3 \text{ cm}^3 = 125 \text{ cm}^3$.

Pay Attention

The product 10×10 is summarized by symbol 10^2 . Similarly, the product $5 \times 5 \times 5$ is expressed by short notation 5^3 . In 10^2 , 10 is called base and '2' is called exponent. 10^2 is read as 'square of 10'. Similarly, 5^3 is read as 'cube of 5'. 10^2 is called exponential form of 100. 5^3 is called exponential form of which number ?

We can write large numbers using exponents in a short form. Normally it is difficult to read, understand and compare large numbers. But expressing numbers in their exponential way becomes easy to read, understand and compare them among themselves.

After multiplying a number by itself repeatedly this brief method of writing the product in exponential form can also be used for letter numbers.

As—

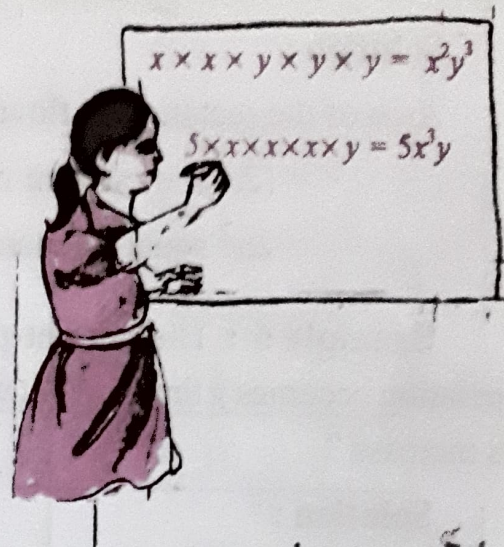
$$x \times x = x^2$$

$$x \times x \times x = x^3$$

x^2 is read as square of x . Similarly, x^3 is read as cube of x . Why so? If the power of x is 1 then we don't write it x^1 . It is written only x .

Similarly,

$x \times x \times y \times y \times y$ is written as x^2y^3 and $5 \times x \times x \times x \times y$ is written as $5x^3y$.



If a is a non-zero integer and m, n are two integers then—

(i) $a^m \times a^n = a^{m+n}$

(ii) $a^m \div a^n = a^{m-n} \ (m > n)$

(iii) $(a^m)^n = a^{mn}$

(iv) $a^0 = 1$.

☞ Try it :

1. Write exponential form of $5 \times x \times x \times x \times x$.

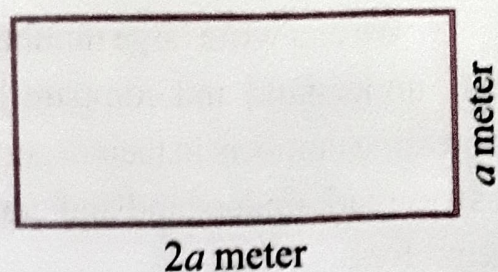
2. Change $m \times m \times m$ in exponential form.

3. What will be the exponential form of $x \times x \times y \times y \times x$?

Example 5. Find out the area of the rectangular floor made in the attached figure.

Solution :

$$\begin{aligned}\text{Area of the rectangular floor} &= \text{length} \times \text{breadth} \\ &= (2a \times a) \text{ square meter} \\ &= 2a^2 \text{ square meter.}\end{aligned}$$



Example 6 : The current population of a village is x . At the end of the year, its population becomes y times. What will be the total population of the village after four years on this increase ?

Solution :

$$\begin{aligned}\text{Current population of village} &= x \\ \text{After 1 year, population of village} &= y \times x \\ \text{After 2 years, population of village} &= y \times (y \times x) \\ &= y \times y \times x \\ \text{After 3 years, population of village} &= y \times (y \times y \times x) \\ \text{After 4 years, population of village} &= y \times (y \times y \times y \times x) \\ &= y \times y \times y \times y \times x \\ &= xy^4.\end{aligned}$$

Hence, the population of the village after 4 years will be xy^4 .

Exercise 5(c)

1. Write following in exponential form :

- (i) $c \times c \times c \times \dots$ 5 times (ii) $5 \times a \times a \times b \times b \times b$
(iii) $7 \times 7 \times 7$ (iv) $t \times t \times y \times y$

2. Write following in product form :

- (i) a^2b^2 (ii) $9ab^3$ (iii) $10x^3y^3z^3$

3. Write following in exponential form :

- (i) $a \times a \times a \times \dots$ n times (ii) $b \times b \times b \times b \times \dots$ n times
(iii) $3 \times 3 \times 4 \times 4 \times a \times a$ (iv) $a \times s \times t \times t$

4. The current income of a man is ₹ a . His income becomes b times every year. What will be his income after 3 years.

5.5 Variable and Constant

Using above-mentioned relation, fill the blanks in the following table :

Diameter of a circle = $2 \times$ radius

$$d = 2 \times r.$$

Table 1

r (cm)	$2 \times r$ (cm)	d (cm)
3	2×3	6
5	 $\times$	10
10	$2 \times$	
r	$2 \times r$	$2r$

In the table mentioned above, we have seen that on changing the values of r , value of d also get changed. i.e., the values of r and d are changed, but 2 does not change.

A car is moving with a speed of 40 km/hr. The relation between distance covered by the car, speed and time is following :

Distance = Speed $\times$ Time

$$s = 40 \times t$$

Use this relation to fill the empty spaces in the following table :

Table 2

Time t (in hr.)	$40 \times t$ (Speed $\times$ Time)	Distance s (in km)
1	40×1	40
2	$40 \times \dots$	80
3	40×3	...
4	40×4	160
t	$\dots \times t$	$40t$

In the above two tables we see that the value of some quantities varies in different circumstances and some do not change the value according to the situation. **The quantities where values change according to the circumstances are called variables and whose values do not change, they are called constant.**

In table 1, r and d are variables and 2 is a constant. Similarly, in table 2, t and s are variables and 40 is constant. Why?

Activity 3

The teacher will ask the students to tell the number of items kept in the closed bag and the number of items placed on the table and make the difference between known and unknown quantities and will tell that we use letters for unknown quantities and their values is not guaranteed. That's why they are called variables and known quantities are called constant because their values are fixed.

Example 7 : Separate the variables and constant in the statement $y = \frac{22}{7}x^2$.

Solution : In $y = \frac{22}{7}x^2$,

Variables : y and x

Constant : $\frac{22}{7}$

Exercise 5(d)

1. Sort out the true and false statements in the following statements :

- (i) x is a constant amount
- (ii) 5 is a constant amount.
- (iii) $(x + 5)$ is a constant amount
- (iv) x^5 is a constant amount.

2. Write the constants in following statements :

- (i) $y = 4x$
- (ii) $y = x + 7$
- (iii) $x + y = 3$
- (iv) $\frac{y}{3} + \frac{y}{4} = 1$

Efficiency Exercise - 5

1. Pay attention to the following mathematical statements and write the number for the letter number in the place of box in your exercise book.

(i) $6 + 4 = x$

$6 + 4 = \square$

(iii) $6 - 2 = a$

$6 - 2 = \square$

(ii) $3 \times 9 = y$

$3 \times 9 = \square$

(iv) $b + 2 = 5$

$\square + 2 = 5$

2. Find out :
 - (i) Number obtained on subtracting x from 10.
 - (ii) Number obtained on addition of $2x$ and $3y$.
 - (iii) 6 times of y .
 - (iv) Number obtained on dividing a by 3.
3. If $a = 5$ and $b = 9$, then find the values of following :
 - (i) $a + 10$
 - (ii) $b - 3$
 - (iii) $a + b - 14$
 - (iv) $a \times b$
 - (v) $30 \div a$
4. Write exponential form of following detailed form :
 - (i) $x \times x \times x \times y$
 - (ii) $q \times q \times q$
 - (iii) $2 \times y \times y \times y$
 - (iv) $5 \times 5 \times 5 \times x \times y \times y \times y$
 - (v) $m \times m \times m \times m$
5. Fill in the blanks $\square$ in your exercise copy :
 - (i) $10 \times 10 \times 10 \times t \times t \times t = 10^{\square} \square^{\square}$
 - (ii) $4 \times 4 \times 4 = 4^{\square}$
 - (iii) $6 \times p \times p \times q \times q \times q = 6^{\square} p^{\square} q^{\square}$
 - (iv) $s \times s \times s \times t \times t = \square^3 \square^2$
6. Write variables and constant in following statements :
 - (i) $5x^2y^2z^3$
 - (ii) $7x^2y^2$
 - (iii) m^4n^2
 - (iv) a^3b^5

In this unit we Learnt

1. The letters, which are used to represent numbers, are called letter numbers. The branch of mathematics in which the problems of letter numbers are solved by using letters, is called Algebra.
2. x, y, z, a, b, c etc. are letter numbers, these numbers do not appear units. Like, if we write length of a room as ' x ', then it is wrong. Writing the length of the room as x meter will be correct.
3. Like numbers, letter numbers and group of letter numbers follow the operations of sum, difference, multiplication and division, and also follow the laws of operations.
4. Like numerical numbers, the letter numbers also have exponential form.

5. The letters of the alphabet, whose numerical value varies in different situations, are called variables.
6. The letter numbers whose numerical value remain constant or unchanged in each and every circumstance, are called constant.

Beginning of Algebra

It is said that algebra as a branch of mathematics began in about 1550 BC. prior to 3500 years ago, when Egyptians started using symbols to express unknown numbers.

It was a very common thing to express the unknown in India by letters around the 300 BC and make an expression. Many great mathematicians such as **Aryabhatta** (born 476 AD), **Brahmagupta** (born 598 AD), **Mahavira** (Who lived in approximately 850 AD) and **Bhaskar-II** (born 1114 AD) and many others contributed a lot in study of Algebra. They have named unknown quantities by letter numbers, alphabet etc. and used first letters of the names of colour to express them like black from 'B', Red from R etc. Indian name for **Algebra** as Beejganit is given by these ancient Indian mathematicians.

Word **Algebra** is derived from the title of 'Algiwar wall Almugawalah' a book written by Muhammad Ibn Al Khawarijm, about a year of about 825 AD by Baghdad's Arab mathematician.

ANSWERS

Exercise 5(a)

1. $3x$ Penciles, 2. (i) $x + 5 = 17$, $x = 12$, (ii) $40 - x = 30$, $x = 10$, (iii) $x \times 6 = 24$, $x = 4$, (iv) $35 \div x = 7$, $x = 5$; 3. $10 - x$, 4. $x + 50 = 120$, 5. $3n$, $4n$

Exercise 5(b)

1. (i) $6 + x$, (ii) $x - 7$, (iii) $5x$ (iv) $\frac{x}{3}$. 2. (i) $d = 2r$, d is the diameter of the circle and r is the radius. (ii) $S = 4x$, S is the perimeter of the square and x is its side. (iii) $A = x \times y$, A is the area of the rectangle, x and y are the length and breadth of the rectangle respectively. (iv) $I = S - C$, I represents the profit, S represents the selling price and C represents the cost price. 3. (a) $(50 - x)$ mangoes, (b) $(100 - x)$ students. 4. $(2x + 2y)$ metre, 5. ₹ $12x$ Paise, 6. $\frac{x}{4}$ girls, 7. $4a$ cm., 8. $(x - 6)$ years, 9. $(16 - x)$ laddoos.

Exercise 5(c)

1. (i) c^5 (ii) $5a^2b^3$ (iii) 7^3 (iv) t^2y^2 ; 2. (i) $a \times a \times b \times b$, (ii) $9 \times a \times b \times b \times b$,
(iii) $10 \times x \times x \times x \times y \times y \times y \times z \times z \times z$; 3. (i) a^n (ii) b^n (iii) $3^2 \times 4^2 \times a^2$ (iv) ast^2 ;
4. ab^3

Exercise 5(d)

1. (i) False, (ii) True, (iii) False, (iv) False; 2. (i) 4, (ii) 7, (iii) 3, (iv) $\frac{1}{3}, \frac{1}{4}, 1$

Efficiency Exercise – 5

1. (i) $6 + 4 = 10$, (ii) $3 \times 9 = 27$, (iii) $6 - 2 = 4$, (iv) $10 \div 2 = 5$;
2. (i) $10 - x$ (ii) $2x + 3y$ (iii) $6y$ (iv) $\frac{a}{3}$; 3. (i) 15, (ii) 6, (iii) 0, (iv) 45, (v) 6
4. (i) x^3y (ii) q^3 , (iii) $2y^3$, (iv) 5^3xy^3 (v) m^4 ; 5. (i) 10^3t^3 (ii) 4^3 (iii) $6p^2q^3$ (iv) s^3t^2
6. (i) 5 constant, x, y and z variables, (ii) 7 constant, x, y variables, (iii) m, n variables,
(iv) a, b variables.

