

UNIT-7 GEOMETRICAL CONCEPTS



- Point
- Collinear points (Linear points)
- Concurrent lines
- Properties of a plane
- Closed and open figures

7.1 Introduction :

In the understanding of the entire nature of the universe and various dimensions of its characteristics and explaining to others, there is unique contribution of different shapes. The birth and evolution of geometry was due to the man's love for nature and leanings of his religious sentiments. Man feels of his favourite geometrical shapes starting from the mother's lap. The Jain texts have described mathematics as the "Best Lotus". The word 'geometry' is derived from the Greek word 'Geometrein'. 'Geo' means land and 'metrein' means measuring. The geometrical concepts in the past evolved from the requirements of art, architecture and land measurement. These concepts have been extensively used in the Vedic ages in the form of altars for the sacrifices, their construction, field demarcation, construction of building, temples, lakes, dams, cities etc. and art and architecture. Also, Now in present days, we develop various types of geometric concepts from the objects like box, table, book, lunch box in our school, ball which children play and its use according to their requirements. Here all these things have different sizes. The ruler and pencil which is used in drawing is straight whereas the 'one rupee coin' or CD is circular.

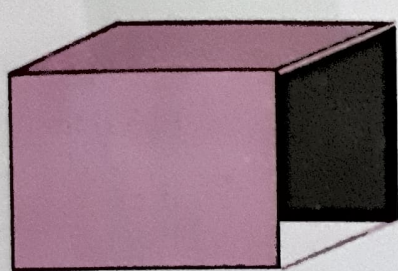
In the opinion of the mathematicians, a point is considered as the basic unit or source of shape of any geometrical construction that has been considered. The great mathematician, Euclid, first undertook an unprecedented task of designing and compiling the theoretical concepts of geometrical shapes in his book 'elements' and laying the foundation of the subject of geometry. These are following :

- A point is one which would have no part.
- A line is widthless length.
- The ends of a line segment are points.
- A straight line is a line that is flat with points on it.

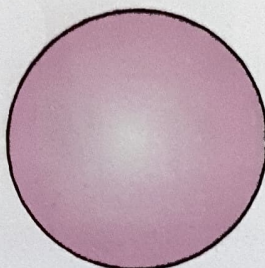
- A plane is one which has length and breadth only.
- A plane surface is a surface that lies flat on itself.

Now, there are a few more interesting facts here that will give you original and detailed information about the various shapes which is present in the environment around you.

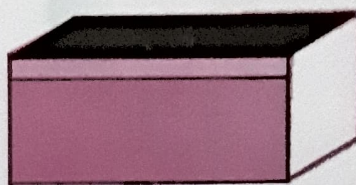
In day to day life we see things of various shapes around us. These shapes are made up of curves or lines. Let's look and think of some of the figures given below.



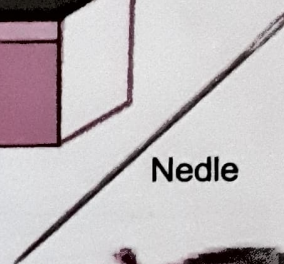
Cuboid



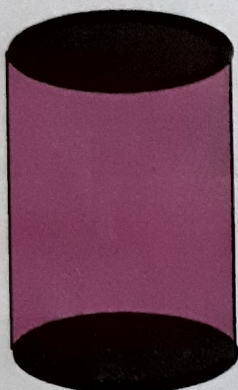
Sphere



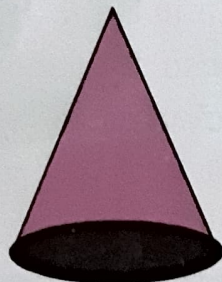
Box



Nedle



Cylinder



Cone



Piece of Stone

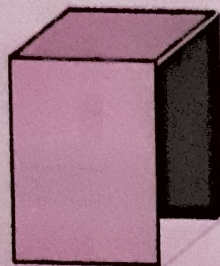


Apple

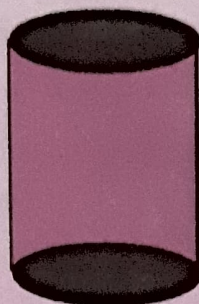
- From looking at the shape of the cuboid and box in the picture one can know that it has 6 plane surface of each which we call face. The four corners of each face are called vertex.
- Look at the shape of the sphere and the apple, there are no clear faces. That is, their surfaces is curved. We call them curved surface or lateral surface.
- The cylinder or cone has no straight edges but their bases are circular. These planes are leveled or flat. Except the surface of their ends, rest surface is curved. The top surface of the stone piece is curved.

Try it

- (a) Write the names of three objects in your environment whose surfaces are:
(1) All plane (2) all curved (3) some are plane and some are curved.
- (b) Tell the name of the surface and their numbers of the following figures:



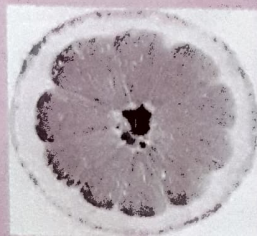
Tea Box



Cylindrical wood



Front of Black board



Half part of lemon



Ball

Discuss and draw conclusions:

Every object has a face or surface. These are of two types:

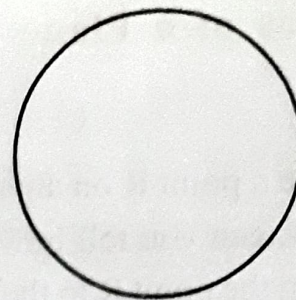
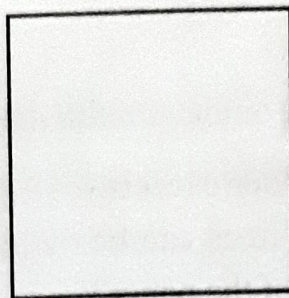
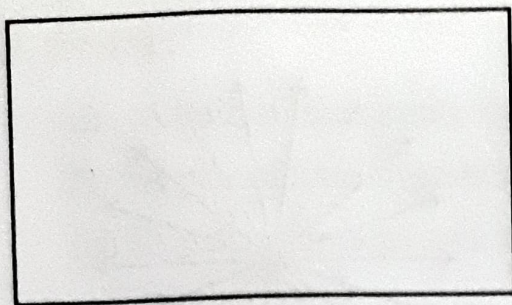
- (1) Plane surface
- (2) Curved surface

- Flat surface is called plane surface.
- Arcual surface is called curved surface.

Activity :

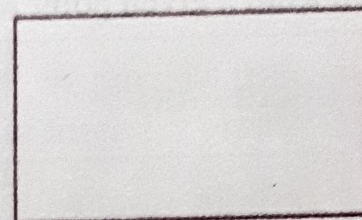
Place a cuboid, cube and a rupee coin on your answer book and make pencil marks and think about the shapes form from them. Here, we see that the rectangle, square and circle are formed respectively from them. All these shapes are on the same plane of the paper. That is, the plane of these shapes are on a larger plane of the paper. The plane of the paper is unlimited spread in all directions. Therefore, we see that a plane is spread unlimited in all directions.





Activity :

Take a piece of paper and fold it from the middle. Take two points A and B on the crease. How many plane can be made with twisted paper, now move both sides of the paper around the fold while keeping the paper crease fixed. We find that the plane of the paper has different positions and each such position represents a plane surface. As the paper moves around, it denotes a new plane surface. As a result, by rotating the paper around the crease infinite number of planes can be formed.

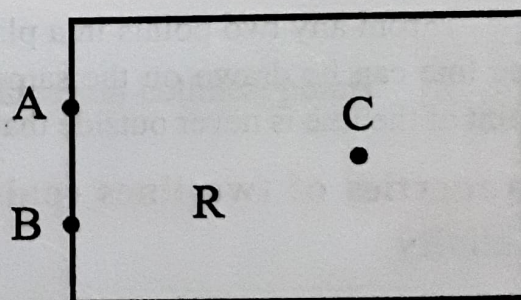


✌ Conclusion:

Innumerable plane surface can be obtained through two points.

7.2 Representation of plane surface:

At least three non-collinear points are required to represent a plane. From the above activities we find that on rotating the paper along the two points A and B folding the paper, are formed separate planes in various positions. If we take point C, non collinear with points A and B then the position of the paper becomes fixed. In such case the paper will represent only one plane as evident from the given picture.



✂ Conclusion:

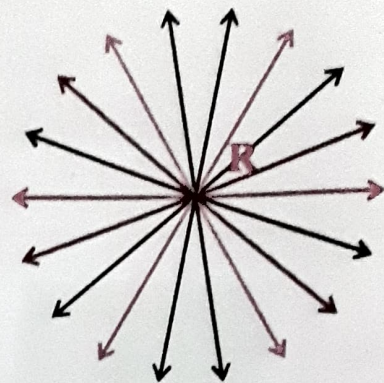
One and only one plane can be drawn through three non-collinear points.

Properties of a Plane

Activity:

Take a point R on a page of your exercise book. Now, can you tell how many lines can be drawn from the point R to the plane of the paper?

Infinite (innumerable) number of lines can be drawn from a given point R. (as in the diagram)



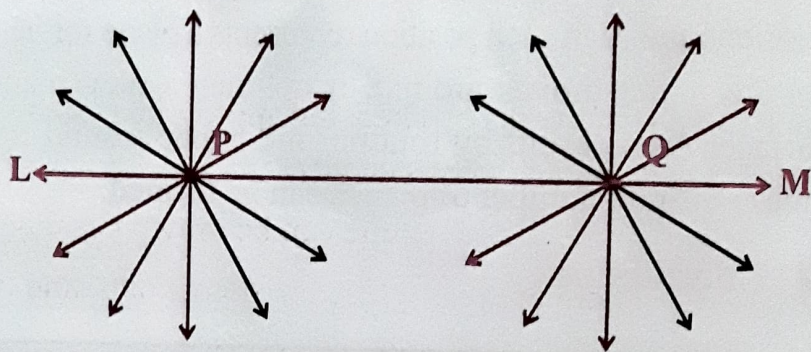
✌ Conclusion:

Infinite number of lines can be drawn through a single point of a plane.

Activity:

Take two points P and Q on the plane of a paper. Here, infinite number of different lines can be drawn from point P and point Q.

The diagram again indicates that one and only one line can be drawn passing through both points P and Q together.



Think and draw conclusion:

From any two points in a plane, one and only one line can be drawn on the same plane, and each point of the line is never outside that plane.



Properties of two lines (pair of lines) in a plane:

Activity

Ask learners of the class to draw different pairs of lines on any page on their answer book. There will be following situations :

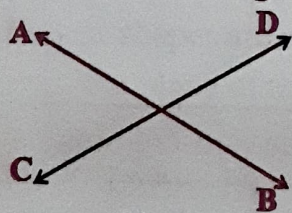


Fig. 1

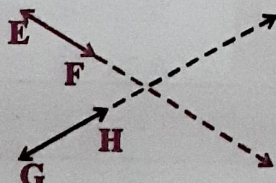


Fig. 2

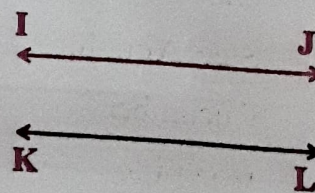


Fig. 3

Let us see:

- (i) A pair of lines intersects each other at some point (Fig 1).
- (ii) A pair of lines intersects each other after extension (Fig 2).
- (iii) A pair of lines pair do not intersects in some case. (Fig 3) (such lines are called parallel lines.)

✂ Conclusion:

Lines on the same plane are either intersecting or parallel. But, it is possible that, the lines in different planes may not intersect even if they are not parallel.

Exercise 7 (a)

1. Fill in the blanks by choosing appropriate word from the following:
(infinitely many, one, curved)
 - (a) line can be drawn passing through two points lies on a plane.
 - (b) There are points on a line lies on a plane.
 - (c) The surface of the sphere is
 - (d) numbers of lines can be drawn from a point lies on a plane.
2. Mark (✓) in front of correct statement and mark (X) in front of wrong statement of the following:
 - (a) The surface of a paper is plane. ()
 - (b) Infinite number of planes can be drawn from three non-collinear points. ()
 - (c) Two lines lying on a plane are always parallel. ()
 - (d) The surface of wall of the room is plane. ()
3. Match the columns A and B with correct pairs:

Column A	Column B
1. Globe	
2. Steel Almirah	P. Plane
3. Cucumber	
4. Iron pipe	Q. Plane and curved
5. Cone	

6. Surface of blackboard

R. Curved

7. Solid cylinder

8. Glass ball

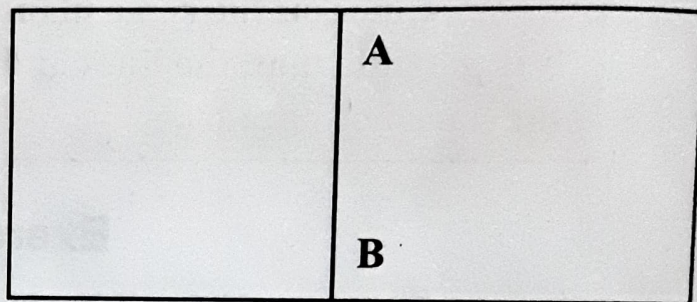
4. How many plane and curved surfaces are there in a solid cylinder?

5. Tell three properties of the plane.

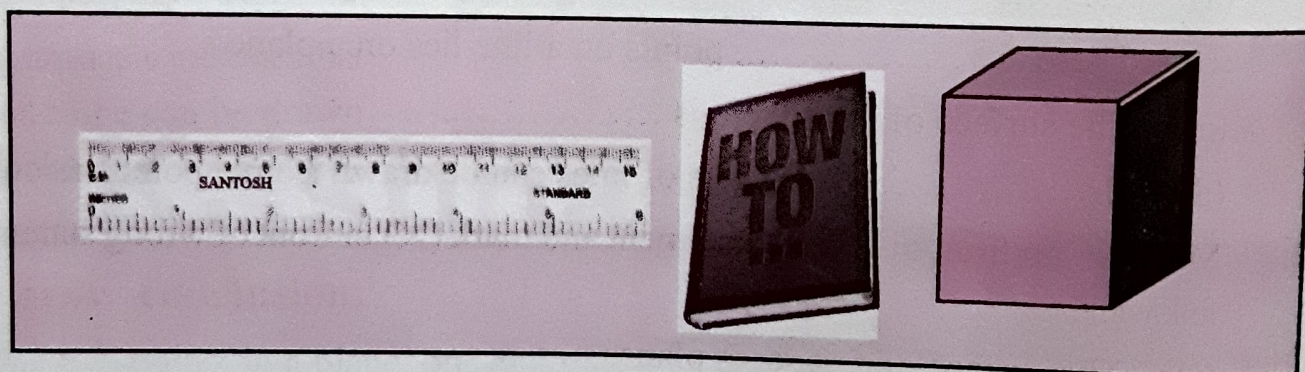
6. Give four examples of objects with plane surfaces and objects with curved surfaces.

7.3 Line Segment :

Take a paper sheet, fold it and press well. Do you see a folding mark on the paper? As shown in the figure, on paper 'AB' appears to be a crease, representing a line segment with A and B having two end points. The line segment of the image is expressed by $\overline{AB}$ or $\overline{BA}$.



Take a thread and pull it with your two hands on the plane so that it is perfectly straight, this position of the thread represents a line segment, and the ends holding with the hands are the endpoints of this line. Line segment AB is denoted by either $\overline{AB}$ or $\overline{BA}$.

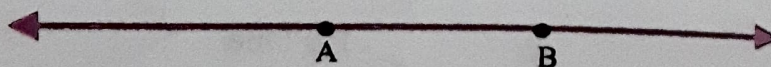


Try it :

Mention names of parts of above shapes that gives glimpse of a line segment.

Line :

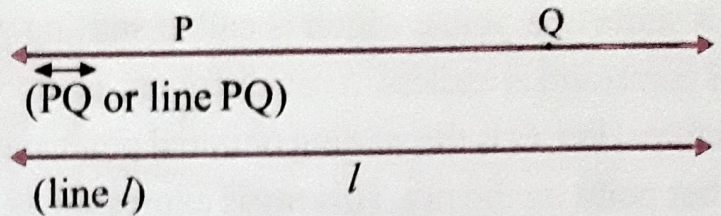
Imagine that the following line segment ($\overline{AB}$) are extended from A in one direction and from B in another direction without any end.



Pay Attention:

Both ends of line $\overleftrightarrow{AB}$ have symbol of an arrow which shows that: the line has no terminal points. It extends infinitely in both directions. There are two methods to represent

the line: if two points P and Q are given on a line, the line is denoted by $\overleftrightarrow{PQ}$ and if the points are not given then it is denoted by a small letter of the English alphabet. As line l , line m , line n ,, etc.

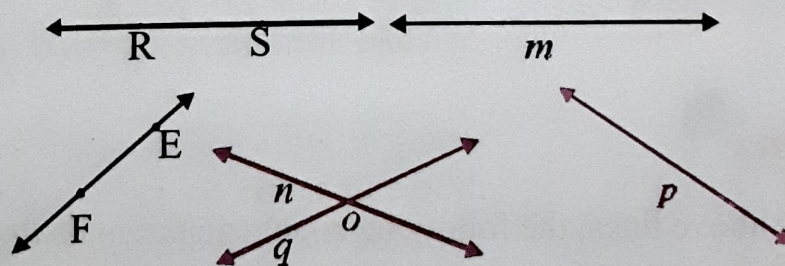


Conclusion:

1. The line is straight which has infinite extension in both opposite directions.
2. There is no end point in a line.
3. A part of a line is called line segment, which has end-points in both sides.
4. In geometry, a line means the whole line, not the part of it.

Try it

Some lines are drawn below. Name them-

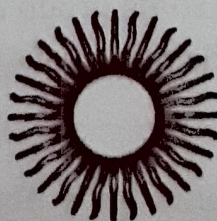


Ray:

See the following figures and think.



Light rays coming from deepak

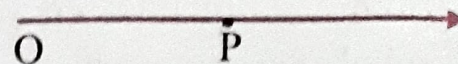


Sun rays coming from sun



Light rays coming from torch

It is clear from the picture that a ray is a part of the line. It starts at a point, which is called starting point (initial point) and is endless in one direction, as shown in the picture. Here O is the starting or initial point and P is the other point on the ray. This ray is expressed as $\overrightarrow{OP}$.



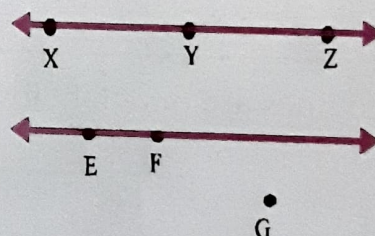
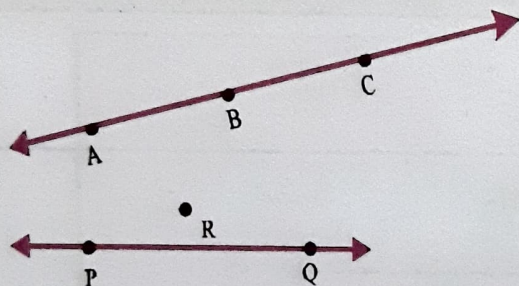
Try it :

Draw three rays whose initial point is O.

7.4 Collinear Points (linear points):

Discuss and draw conclusions:

1. Mark following group of three points on the plane of the paper, (i) A, B and C (ii) X, Y and Z (iii) P, Q and R (iv) E, F and G as follows. Draw lines from any two points on each group, as (i) A and B (ii) X and Y (iii) P and Q and (iv) E and F.



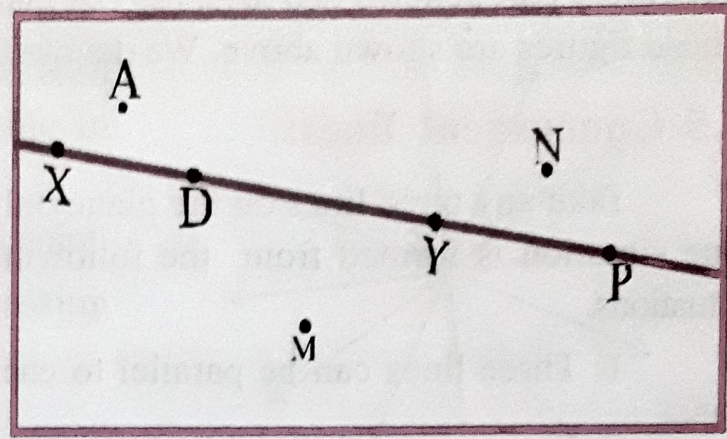
Pay Attention:

1. On observing the above lines, the following conclusions can be drawn:
 - i. The three points of the some group are on the same line i.e. the line goes through the three points of the group. Such as a line goes through group A, B, C and another line from group X, Y, Z.
 - ii. The three points of the some group are not on the same line. Such as group P, Q, R has point R outside the line PQ and points in the group E, F, G has point G outside the line EF.

Three or more points on a plane are called collinear (a linear points) if all are situated on the same line.

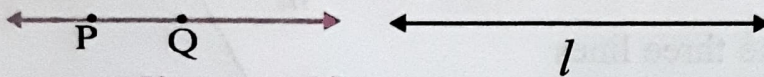
Activity:

Draw points A, D, M, N, P, X and Y on the plane of the paper as shown in the figure. Fold and press it in such a way that points X and D lie on the fold. After opening the paper, the position of the other points becomes clear as to whether it is located on this line. We find that the points X, D, Y and P are located on the line. So these are collinear points while points A, N, and M are beyond this line. Clearly A, D, M, N, P, X and Y are the non-collinear points.



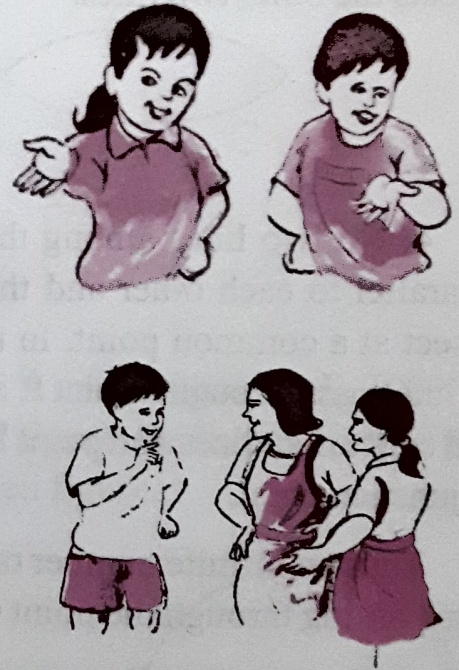
Pay Attention:

The two ends have marks of the tip of an arrow which shows that there is no terminal point on the line. It extends to the infinite in both directions. This line is represented by PQ or small letter l of English.



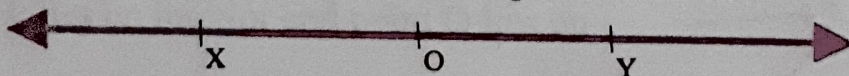
What have we discussed?

1. The line is straight, the extension of which is infinite in both directions.
2. The line has no end points. If the line has two points A and B, then the line is represented by line AB or $\overleftrightarrow{AB}$.
3. The line can be denoted by in the English alphabet by writing a small letter like l , m , n etc.
4. A part of a line is called a line segment having an end point on either side of it. If the end points of the line are points A and B then it is represented as AB.
5. In geometry, we mean the line, as the whole line, not any part of it.



Do it, Discuss and Conclude:

Draw a line XY on the paper and mark a point O on it.



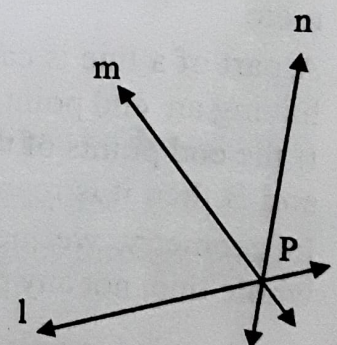
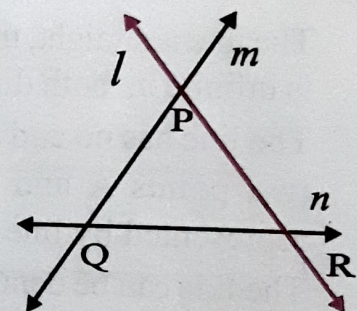
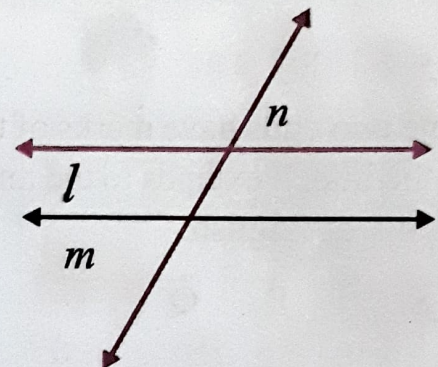
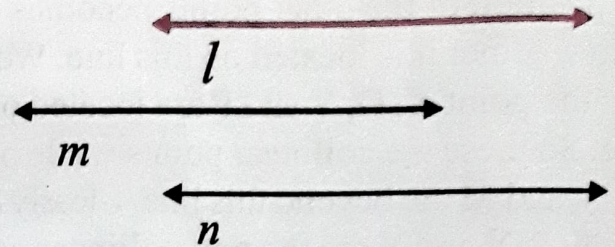
We find that the line is divided into two parts. One part of the line contains the point O and all the points of the line on the left side of the point O and the second part of the line

which has the point O on it and all of the points in the line lies on the right side of point O. Both these part denotes two rays, ray OX and ray OY. Point O is called initial point of them. These figures are shown above. We denote them by $\overrightarrow{OX}$ and $\overrightarrow{OY}$ respectively.

7.5 Concurrent lines:

Take any three lines on the plane only one situation is formed from the following situations.

1. Three lines can be parallel to each other.
2. Two of the three lines are parallel to each other and the third intersects them.
3. No two lines among the three lines are parallel to each other, each of which intersects the other two lines.



(concurrent lines and intersection point P)

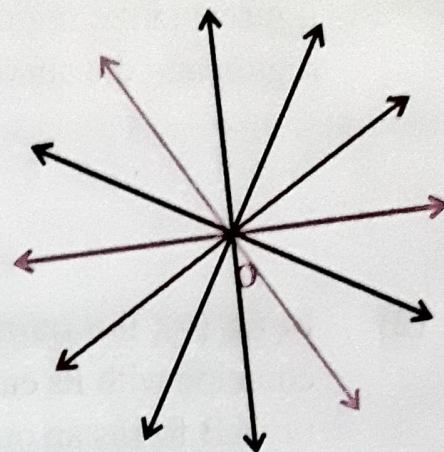
4. No two lines among the three lines are parallel to each other and the three lines intersect at a common point. In this case, the three lines pass through a point P. Such lines are called concurrent lines and point P is known as their junction point.

Thus an infinite number of lines can be drawn passing through the point O.

If three or more lines pass through the same point on the plane, then they are called a concurrent lines and this point is called their point of intersection.

Discuss in group for the Truth and Falsity of the following sentences:

- A line is of finite length.
- Two lines on a plane always intersect each other. Whereas, it is not possible in different planes.
- Only one line can be drawn from a point.
- Only one line can be drawn passing through two points.
- There are two directions in a line.
- Two parallel lines never meet each other.
- Three or more lines on a plane are concurrent if all the lines pass through a point.



concurrent lines and intersection point P

7.6 Perception of closed and open Figures:

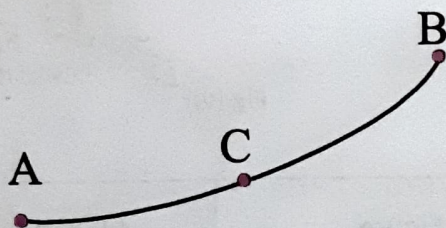


Fig. (i)

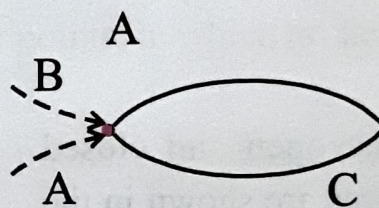


Fig. (ii)

Here in fig (i), ACB is a curve whose starting point is A and end point is B. If the point A and B coincides (fig (ii)) then curve ACB is called closed figure. On the contrary, if points A and B do not coincides then the figure ACB is called open figure.

Interior and Exterior region in closed figures:

In fig (iii), ACB is a closed figure, in which points A and B coincides. On moving anticlockwise from point A to point C, the region on the left is called interior region and the region on the right is called exterior region. In fig (iii), point P lies on interior and point Q lies on exterior region.

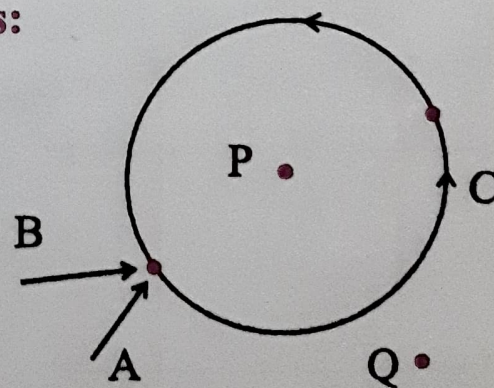


Fig. (iii)

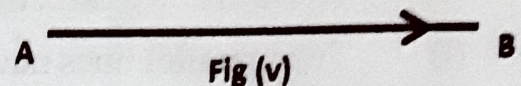
Points A, C and B lies on the curve. Thus,

- (1) In fig (iv), the shaded region is a closed figure and the region except the interior region and the curve is exterior region.



Exterior
Fig (iv)

- (2) In fig (v), the starting point A does not coincide with its end point B. Therefore, ray AB forms an open figure.



- (3) Fig (vi) represents the projection of a bird. As the end points A and B coincide, the region ACDEB is a closed region. In fig (vi), P is interior and K is any exterior point.



Fig (vi)

- (4) Some open and closed figures are shown in the adjacent table.

Open Figure	Closed Figure
 Line	 Triangle
 Angle	 Quadrilateral
 Open string	 String with knot

Exercise 7 (b)

1. Draw a line in your exercise book and write its name by using a small letter of English alphabet.
2. Give three examples with the help of available things around you, which represents a sense of a line (or its part).
3. Differentiate between a line and ray by drawing a figure.
4. Draw a ray passing through given initial point A and from other point B.

Efficiency Exercise 7

1. How many lines can be drawn passing through three collinear points?
2. Write down the following points on the plane of the paper and clarify:
 - (a) How many lines can be drawn from a point?
 - (b) How many lines can be drawn from two points?
 - (c) How many lines can be drawn from three points, which are not collinear, taking two at a time?
 - (d) How many lines can be drawn from four points, in which no three are collinear, taking two at a time?
 - (e) How many lines can be drawn from five points, in which no three are collinear, taking two at a time?

3. Match the correct pairs in column A and column B:

Column A

Column B

(a) Ray

$\overleftrightarrow{AB}$

(b) Line

$\overline{AB}$

(c) Line segment

$\overrightarrow{AB}$

(d) Closed figure

ABA

4. How many points can be there in a line lying on a plane?

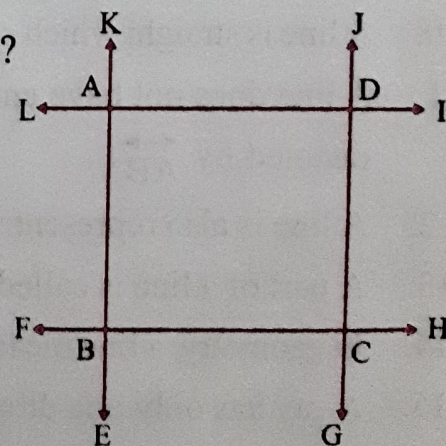
5. (i) Write the name of lines in the given figure.

(ii) Write the name of those lines which passes through point A in the figure.

(iii) Write the name of closed figure in the figure.

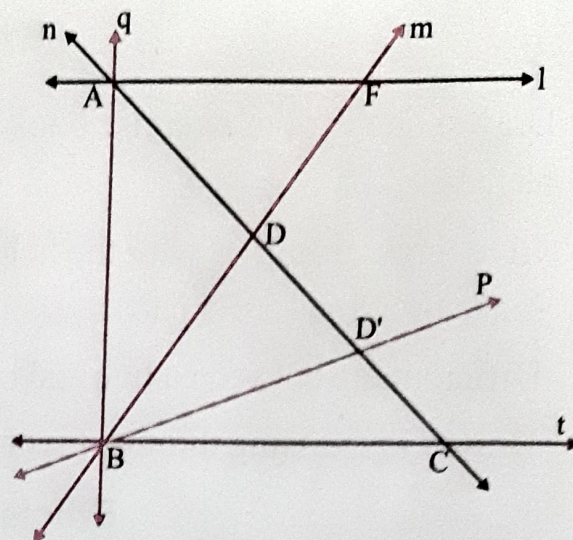
(iv) Which type of figure KADJ is? Open or closed?

(v) Figure KAL is open or closed?



6.

- (i) Write the concurrent lines and their point of intersection in the following figure.
- (ii) In the adjacent figure, figure AFB is open or closed?
- (iii) In the adjacent figure, how is the region FDD'P is and is it open or closed?



Project: There is one and only one plane passing through three non-collinear points. Where it can be used in our daily life? Tell by observing equipments in objects in your environment and the school.

In this unit we learnt about :

1. Every solid has a face. The face is called surface.
2. Every object in the surrounding has a surface.
3. Surfaces are of two types:
 - (i) Flat (ii) Curved
4. The plane spreads out infinitely in all directions.
5. There are infinitely many planes passing through two points.
6. One and only one plane can be drawn from three non-collinear points.
7. One and only one line can be drawn from the two points lying on a plane. This line lies on the same plane in which the points lie.
8. Every point of the line passing through the two points lying on the plane, are on the same plane. No point of the line lies outside the plane.
9. Two lines lying on the same plane either intersects or parallel.
10. A line is straight which spreads infinitely in both directions.
11. A line does not have end points. If points A and B are taken on a line then the line is denoted by $\overleftrightarrow{AB}$.
12. A line is also represented by the small letters of English alphabets like l, m, n etc.
13. A part of a line is called line segment, which has end points on both sides.
14. In geometry, a line means the whole line, not the line segment.
15. A ray has only one direction. It is represented by arrow ($\rightarrow$)

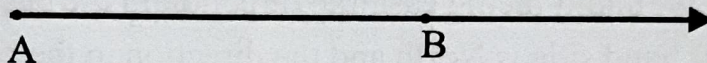
16. A ray has only one end point or initial point.
17. Three or more points on a plane are called concurrent if they all lie on the same line. This line is called line of concurrence.
18. The point, where two lines lying on a plane intersect, is called intersection point.
19. If three or more lines pass through the same point on the plane, then they are called a concurrent lines and this point is called point of intersection.
20. An open, or closed figure can be formed by a curve.

ANSWERS

Exercise 7 (a)

1. (a) One (b) Infinite (c) Curved (d) Infinite; 2. (a) ✓ (b) ✗ (c) ✗ (d) ✗ 3. 2 and 6 → P, 5, 7 → Q and 1,3,4 and 8 → R; 4. 2 plane and one curved.

Exercise 7 (b)

3. line $\leftrightarrow$, ray $\rightarrow$; 4. (d) 

Efficiency Exercise 7

1. One; 2. (a) Infinite (b) One (c) three (d) 6 (e) 10; 3. (a) ray $\overrightarrow{AB}$, (b) line $\overleftrightarrow{AB}$, (c) line segment $\overline{AB}$ 4. Infinite 5. (i) $\overleftrightarrow{AD}$, $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$, $\overleftrightarrow{CD}$ (ii) $\overleftrightarrow{AD}$, $\overleftrightarrow{AB}$; (iii) ABCD, (iv) Open, (v) Open; 6. (i) Point of concurrent A, concurrent lines l, q, n and point of concurrent B, concurrent lines q, m, p, t , (ii) Closed (iii) Open.

