

UNIT-9

PERPENDICULAR AND PARALLEL LINES



- Concept of perpendicular and parallel lines
- Name and property of the angles formed by two lines and a transversal line.
- Draw a parallel line with the help of scale and set square.
- With the help of the characteristics of parallel lines, it is possible to measure unknown angles and draw parallel lines to verify them.

9.1 Introduction

You have read about plane, point, lines and angles. Now we will study about the relationship between lines. You have seen the use of thread and plumb bob by mason for the smoothness of wall at the time of construction of house. Are you think what he see by plumb bob and thread ? He decide the perpendicularity of the wall from the surface by plumb bob, whenever he inspects the parallel ness of each layer of breck of the wall. Come, now we will study about the perpendicular and parallel lines.

9.2 Perpendicular lines :

The edge of each page of your book represent that two lines are perpendicular to each other, you can see in the figure (i)

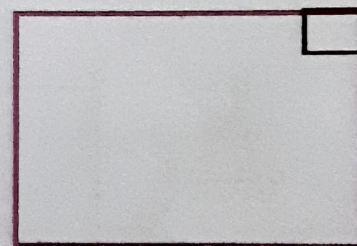
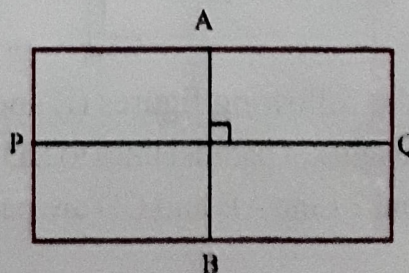
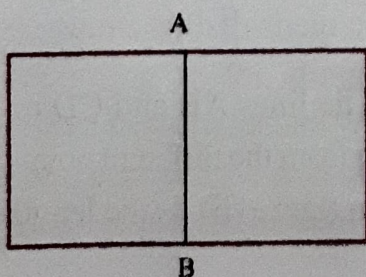


Fig. (i)

Do it :

Take a page of notebook. Fold it by middle and make a crease AB of this fold. Now you fold it in an another direction again that the one part of AB has covered the second part completely and made the crease PQ of this fold. Now open the paper, you observe that both creases of the fold are perpendicular to each other, that is AB and PQ are perpendicular to each other.



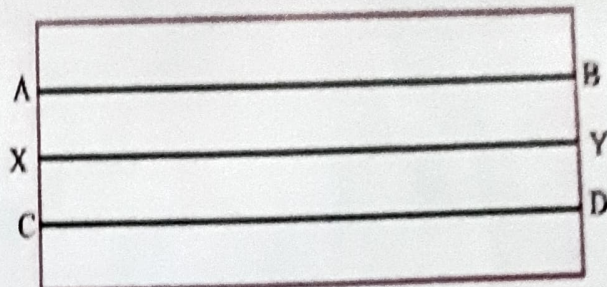
If two lines intersect to each other and the angle between them is right angle, then these lines are called perpendicular lines to each other, we write it $AB \perp PQ$.

Try it :

Draw a perpendicular from a point of a given line on the paper.

9.3 Parallel lines :

Take a page of notebook, fold it from middle and make the crease by XY. Again fold it in the same direction, make the crease AB and CD. See that you have find the lines according to the figure.



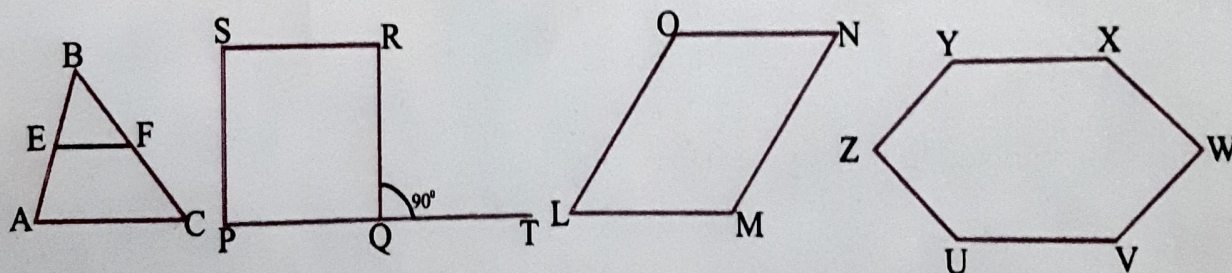
Here we observe that the distance between any two lines is always equal. They do not intersect to each other however if we extend it.

Those lines in a plane which are at equal distance always and they do not intersect, are called parallel lines. It is written $\overline{AB} \parallel \overline{CD}$.

Try it :

Fold a paper and make a line parallel to this fold.

Dot it :



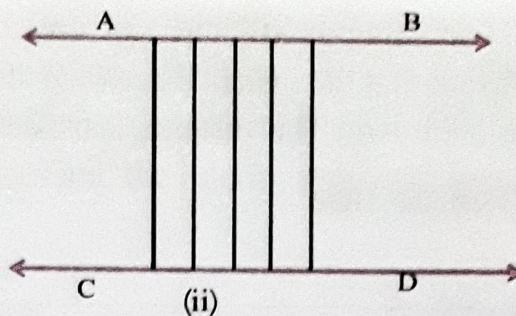
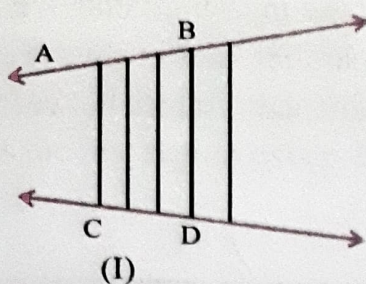
Write down pair of parallel and perpendicular line segments from above figure.

9.4 To find the distance between two parallel lines :

Do it, think and find conclusion :

Activity :

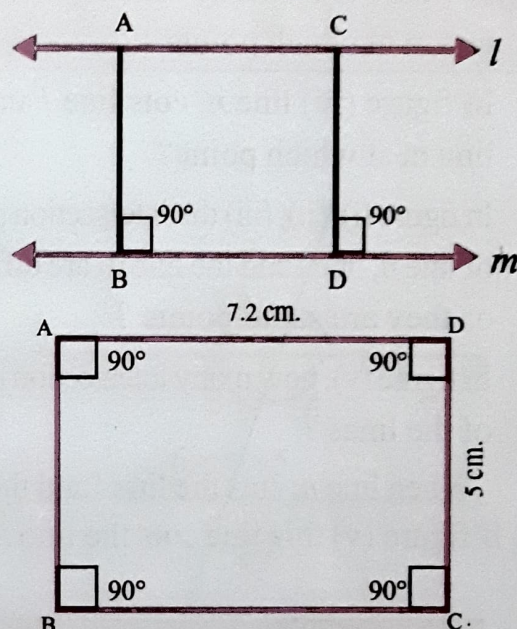
Observe the following figures (i) and (ii). The lines AB and CD of figure (i) are not parallel. Are the lengths of parallel lines to BD joining from the different points of AB to different points of CD equal ? Line AB and CD are parallel in figure (ii). Is the length of perpendicular



drawn from different points of AB to the lines CD are equal? We get that the lengths of different parallel lines to BD in figure (i) are not equal. Whenever the lengths of different drawn perpendicular lines in figure (ii) are equal. The length of this perpendicular for the parallel lines is called perpendicular distance or distance.

Exercise 9(a)

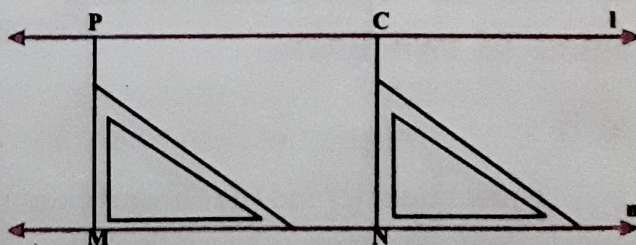
1. Give four examples of pairs of parallel lines of objects in your neighbourhood.
2. l and m are two parallel lines in the figure. AB and CD are perpendicular distance between these lines. If $AB = 3$ cm, find length of CD.
3. The measurements of angles of rectangle ABCD have written in the figure.
 - (i) Is $AB \parallel CD$? If 'yes' then why ?
 - (ii) Is $AD \parallel BC$? If yes then why ?



To find the distance between the parallel lines

Do it, think and find conclusion :

- Draw two parallel lines in the direction of its edges with the help of scale.
- Take two points M and N on line m .
- Draw perpendiculars MP and NC from these points with the help of set square according to the diagram. Measure these perpendiculars. What is the relation between the distance in the direction of perpendiculars MP and



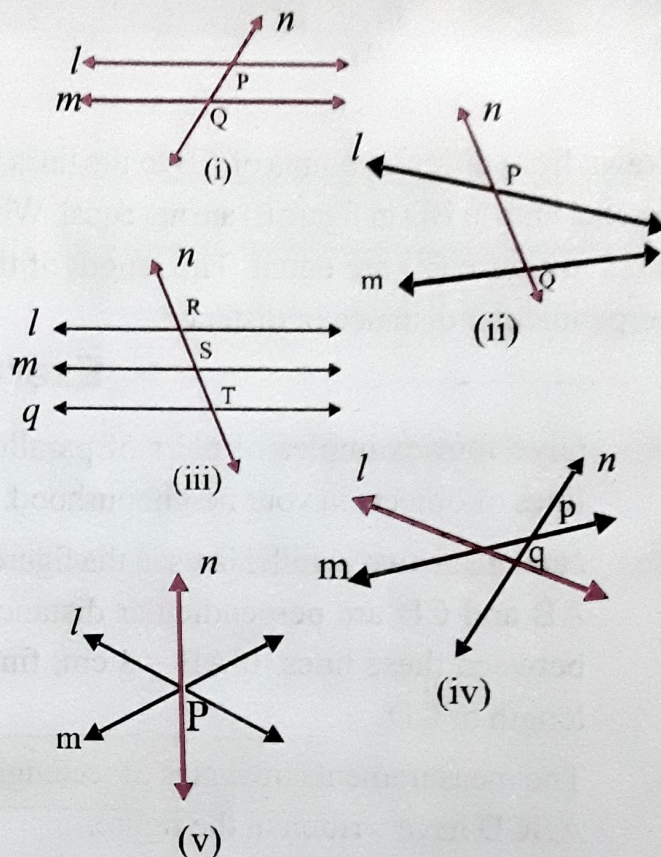
NC ? The value of MP and NC are equal by the measurement. We reach at the conclusion that the distance between any two parallel lines is always equal at each place, in this way. This distance is called perpendicular distance between these lines.

9.5 Transversal line

Do it :

See the following figures and to say

- In figure (i), line n cuts line l and the line m at which points ?
- In figures (ii) line n cuts line l and the line m at which points ?
- In figure (iii) line n cuts line l and the line m at which points ?
- In figure (iv) line n cuts line l and the line m at which point ?
- In figure (i), (ii), (iii) the intersection points of line n , line l and the line m are different or they are same points ?
- In figure (v), how many intersection points of the lines ?



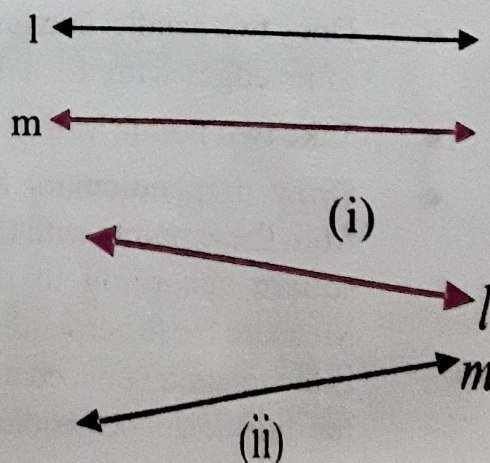
When line n , cuts the line l and the line m at different points then it is called a transversal line. If figure (v) this line cuts the line l and the line m at same point, so it is not a transversal line.

Transversal line, is that line which cuts two or more than two lines at different points.

9.6 Interior region and exterior region of two lines.

Do it :

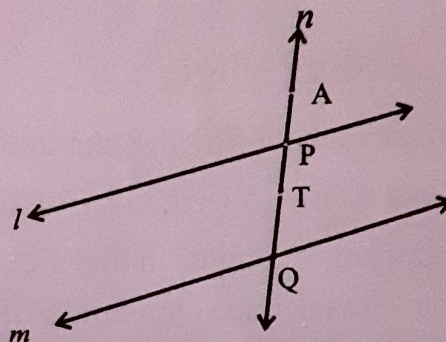
Draw figure (i) and (ii) on your exercise book. Colour the region between the lines l and m by pencil.



The coloured region by pencil between the lines is Interior region rest region which are not between the lines l and m , is the exterior region. Therefore, we can say that the interior region of two lines is the total region between them except these lines, and the exterior region of two lines is the rest region except these lines and the interior region between these lines.

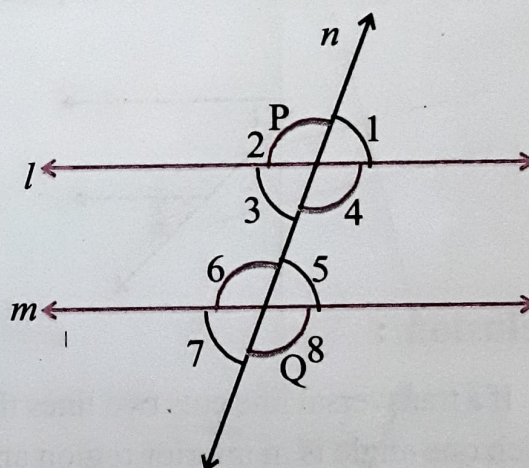
Try it :

Show the points A and P, on the line as shown in the adjacent figure. What is the name of that line on which point A and T lie? Is point A lies in the exterior region of the lines l and m ? Is point T lies in the interior region of the lines l and m ?



9.7 : The angle made by transversal line : Do it, think and discuss :

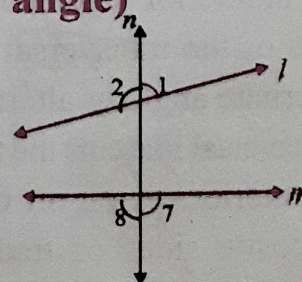
- (i) Draw two lines l and m
- (ii) Draw a transversal line, which cuts the lines l and m .
- (iii) Represent the intersection points by P and Q, which is shown as in the figure.
- (iv) Represent the angles 1, 2, 3, 4, 5, 6, 7 and 8, made by the transversal line n and two lines l and m as shown in the figure.



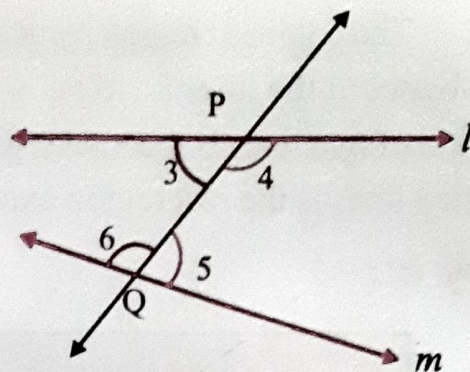
There are eight angles formed when a transversal line cuts two lines. Four angles are in out side region and rest four angles are in inside region.

9.8 : Exterior angle, Interior angle, corresponding angle, alternate angle (alternate interior angle and alternate exterior angle)

Exterior angle : Observe in the adjacent figure, line n cuts the lines l and m . The angles 1, 2, 7 and 8 lie in exterior region of lines l and m and the transversal line n . These angles are called exterior angle.



Interior angle : Observe the angles formed by two lines l and m and the line PQ, these angles are represented by 3, 4, 5, 6. These angles are in inside region. **These angles are called interior angles.** It is also shown that angles 4, 5 are on one side of transversal line and angles 3, 6 are on other side of the transversal line.

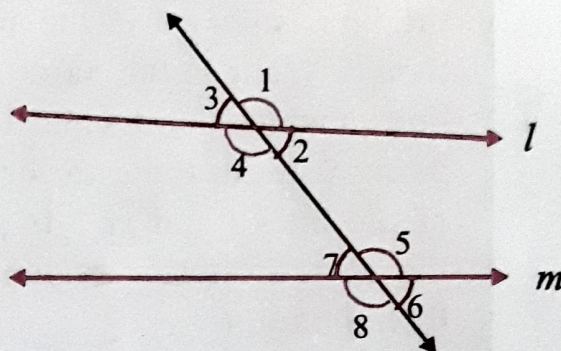


Corresponding angle :

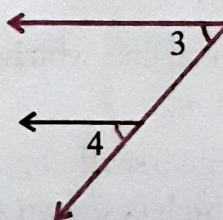
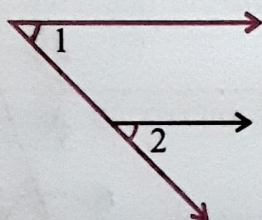
Observe the following pair of angles in figure :

- (i) 4, 8 (ii) 1, 5 (iii) 3, 7 (iv) 2, 6

Each pair of angles in this are on the same side of the transversal line. One angle of each pair is in interior region and other is in exterior region. These are called corresponding angle to each other.



Some other pairs of corresponding angles are given below. Draw these in your note book.

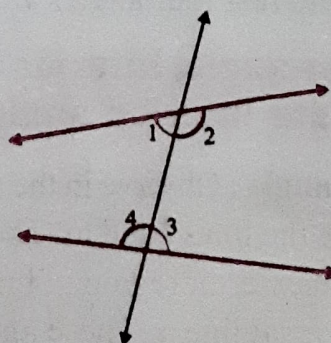


Conclusion :

If a transversal line cuts two lines then two angles on different vertices on the same side in which one angle is in interior region and other is in outer region, are called corresponding angle to each other.

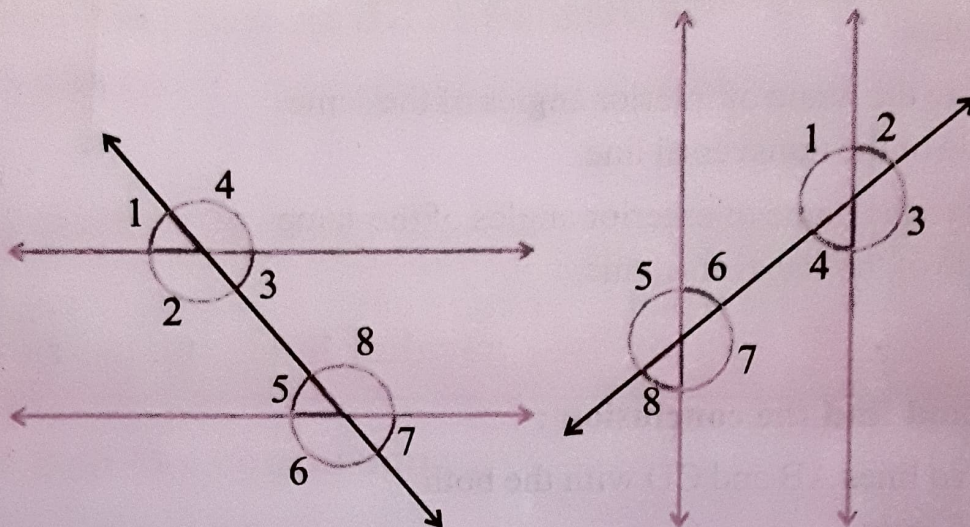
Alternate Interior Angle (Alternate Angle) :

Observe the pairs of angles in adjacent figure, (i) 1, 3, (ii) 2, 4 both angles of each pair are in interior region, but they are on opposite side of the transversal line. These are called alternate angle or alternate interior angle. If a transversal line cuts the two lines, then the angle of interior region on intersecting points and opposite sides of transversal line is called alternate angle.



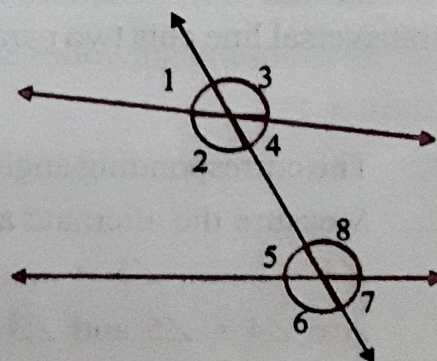
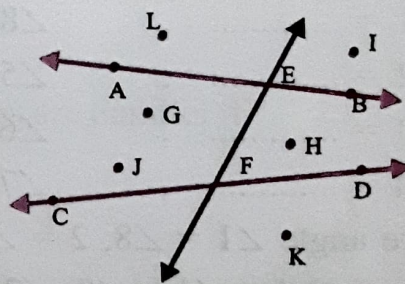
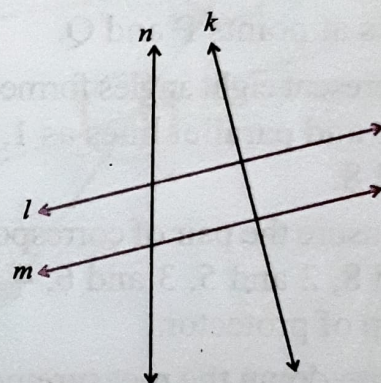
Try it :

Draw the following figures on your note books , determine write the pairs of alternate angles by inspection.

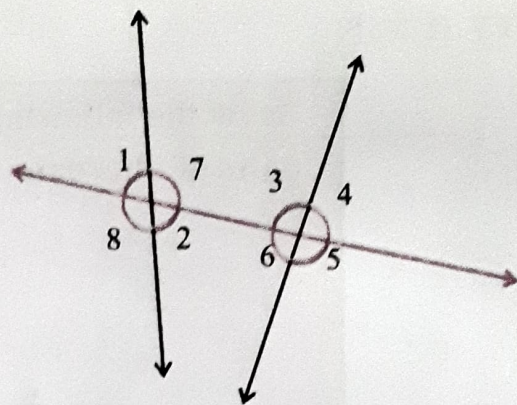


Exercise 9(b)

- Find the transversal lines from the adjacent figure and write it on your note book.
- Write which points are in interior region and exterior region of the line AB and line CD respectively ?
- Observe the adjacent figure and fill in the blanks on your note book.
 - $\angle 2$ and are alternate angle.
 - $\angle 2$ and are corresponding angle.
 - $\angle 2$ and are interior angles of one side of the transversal line.
 - $\angle 3$ and are the angles of one side of the transversal line.



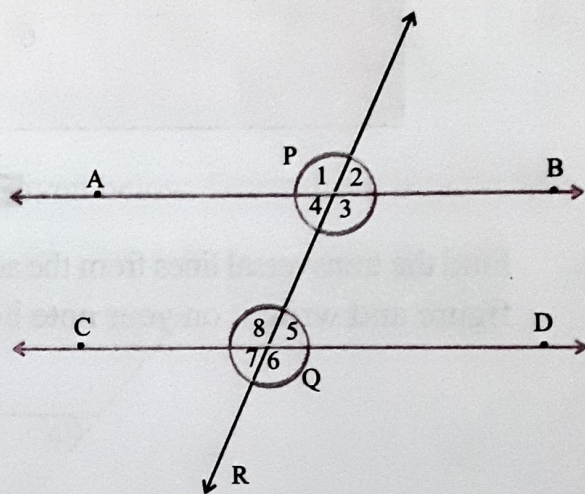
4. Give answer to see the adjacent figure.
- Give the name of pairs of alternate angles.
 - Give the name of pair of corresponding angles.
 - Give the name of interior angles of the same side of the transversal line.
 - Give the name of exterior angles of the same side of the transversal line.



9.10. The angle formed by two parallel lines and a transversal line.

Do it, think and find the conclusion :

- Draw two lines AB and CD with the both edges of scale.
- Draw the transversal line which cuts both lines at points P and Q.
- Represent eight angles formed by transversal line and parallel lines as 1, 2, 3, 4, 5, 6, 7 and 8.
- Measure the pair of corresponding angles 1 and 8, 2 and 5, 3 and 6, 4 and 7 with the help of protector.
- Write down the measurement on your note book.



$$\angle 1 = \dots\dots\dots, \quad \angle 8 = \dots\dots\dots$$

$$\angle 2 = \dots\dots\dots, \quad \angle 5 = \dots\dots\dots$$

$$\angle 3 = \dots\dots\dots, \quad \angle 6 = \dots\dots\dots$$

$$\angle 4 = \dots\dots\dots, \quad \angle 7 = \dots\dots\dots$$

Are angle $\angle 1 = \angle 8$, $\angle 2 = \angle 5$, $\angle 3 = \angle 6$, $\angle 4 = \angle 7$?

We find that $\angle 1 = \angle 8$, $\angle 2 = \angle 5$, $\angle 3 = \angle 6$, $\angle 4 = \angle 7$. Therefore we can say that if a transversal line cuts two parallel lines then the formed corresponding angles are equal.

Conclusion :

- The corresponding angles are equal if a transversal line cuts the two parallel lines.
- Measure the alternate angle $\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 8$.

$$\angle 4 = \dots\dots\dots, \quad \angle 5 = \dots\dots\dots, \quad \angle 3 = \dots\dots\dots, \quad \angle 8 = \dots\dots\dots$$

Are $\angle 4 = \angle 5$ and $\angle 3 = \angle 8$?

Conclusion

The alternate angles are equal, if a transversal line cuts two parallel lines.

3. (i) Measure the interior angle $\angle 3$ and $\angle 5$ and add them. Is $\angle 3 + \angle 5 = 180^\circ$?

(ii) Measure the interior angle $\angle 4$ and $\angle 8$ and add them.

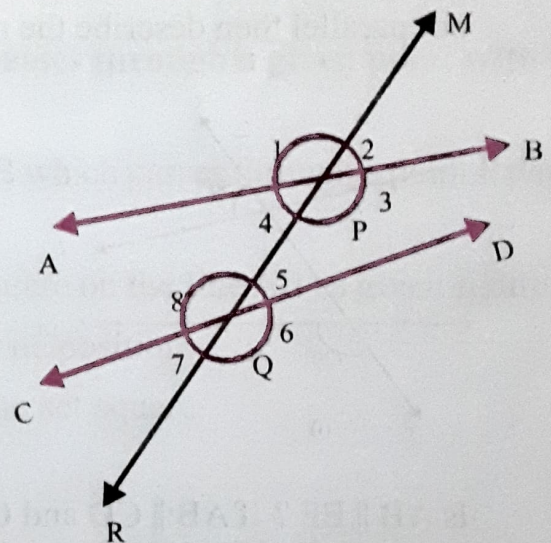
Is $\angle 4 + \angle 8 = 180^\circ$?

Conclusion

If a transversal line cuts two parallel lines, then the sum of interior angles of same side of the transversal line is 180° , That is these angle are supplementary.

Do it, think and find the conclusion

Draw two lines AB and CD on your note book as according to the figure, which are not parallel and draw a transversal line MR which cuts AB and CD at points P and Q. Show the angles made by transversal line and non-parallel lines as 1, 2, 3, 4, 5, 6, 7, 8 and observe that :



- (i) Corresponding angles are equal or not.
- (ii) Alternate angles are equal or not.
- (iii) The sum of interior angles of same side of the transversal line is 180° or not.

Conclusion

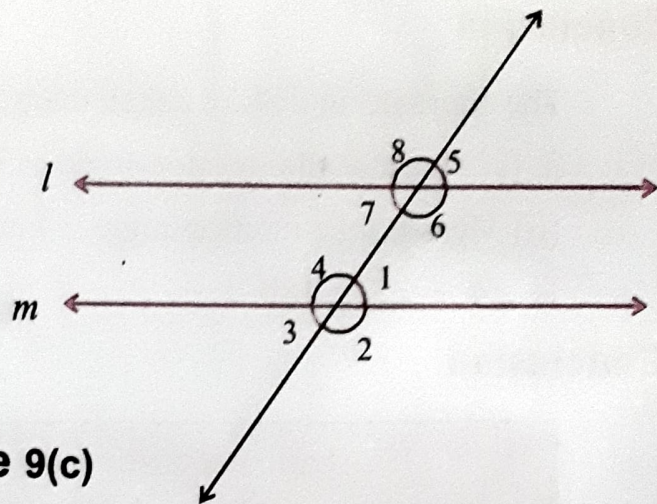
If a transversal line cuts two lines, then these two lines are parallel if

- (i) Corresponding angles are equal, or
- (ii) Alternate angles are equal, or
- (iii) The sum of interior angles of same side of the transversal line is 180° .

To see the adjacent figure and group discussion on the following questions and find the conclusion.

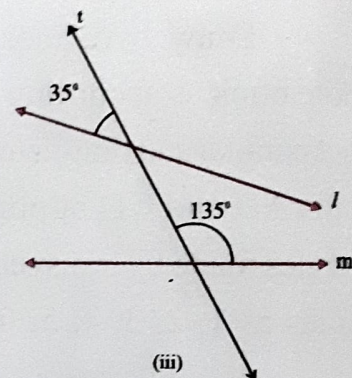
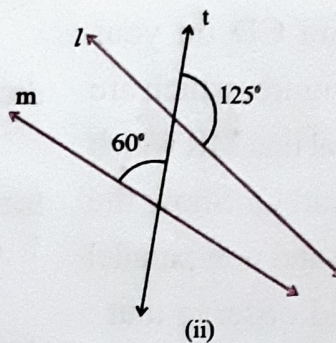
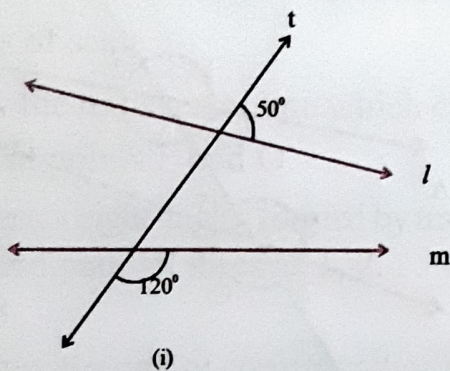
- (i) Write the name of three angles which are equal to $\angle 1$.
- (ii) Write the name of three angles which are equal to $\angle 4$.

- (iii) If $\angle 1 = 70^\circ$, then $\angle 7 = \dots\dots$
 (iv) If $\angle 2 = 100^\circ$, then $\angle 3 = \dots\dots\dots$
 (v) If $\angle 3 = 60^\circ$, then $\angle 5 = \dots\dots\dots$
 (vi) If $\angle 4 = 110^\circ$, then $\angle 6 = \dots\dots\dots$

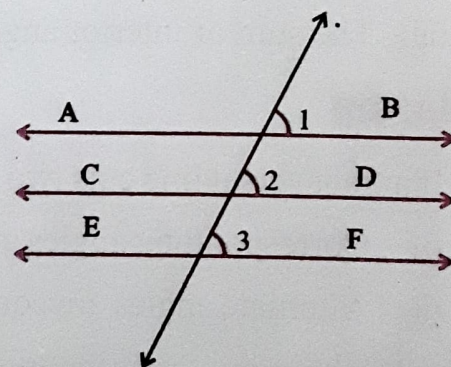


Exercise 9(c)

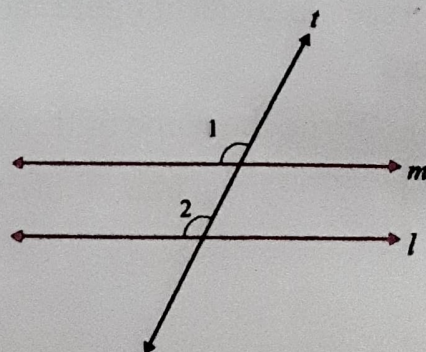
1. In the following figure, a transversal line t cuts two lines l and m . The measurement of some angles are written. Are l and m parallel? If it is, then describe its reason. If it is not parallel then describe the reason also :



2. Is $AB \parallel EF$? If $AB \parallel CD$ and $CD \parallel EF$ in the figure, describe the reason.



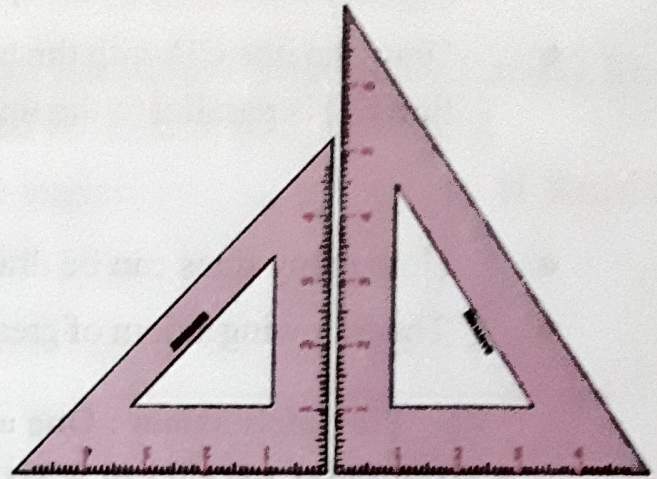
3. $\angle 1$ and $\angle 2$ are equal in the figure. Describe reasons that it can be proved that $l \parallel m$.



9.11 To draw the parallel lines with the help of scale and set square.

Do it and remember the result

Take up the following instruments from the Geometry box which are shown in the figure. These are set square. One angle is 90° and rest two angles are 45° in the first set square. One angle is 90° and rest two angles are 30° and 60° respectively in the second set square. Construct the angles 30° , 45° , 60° and 90° with the help of these and measure it with protector.

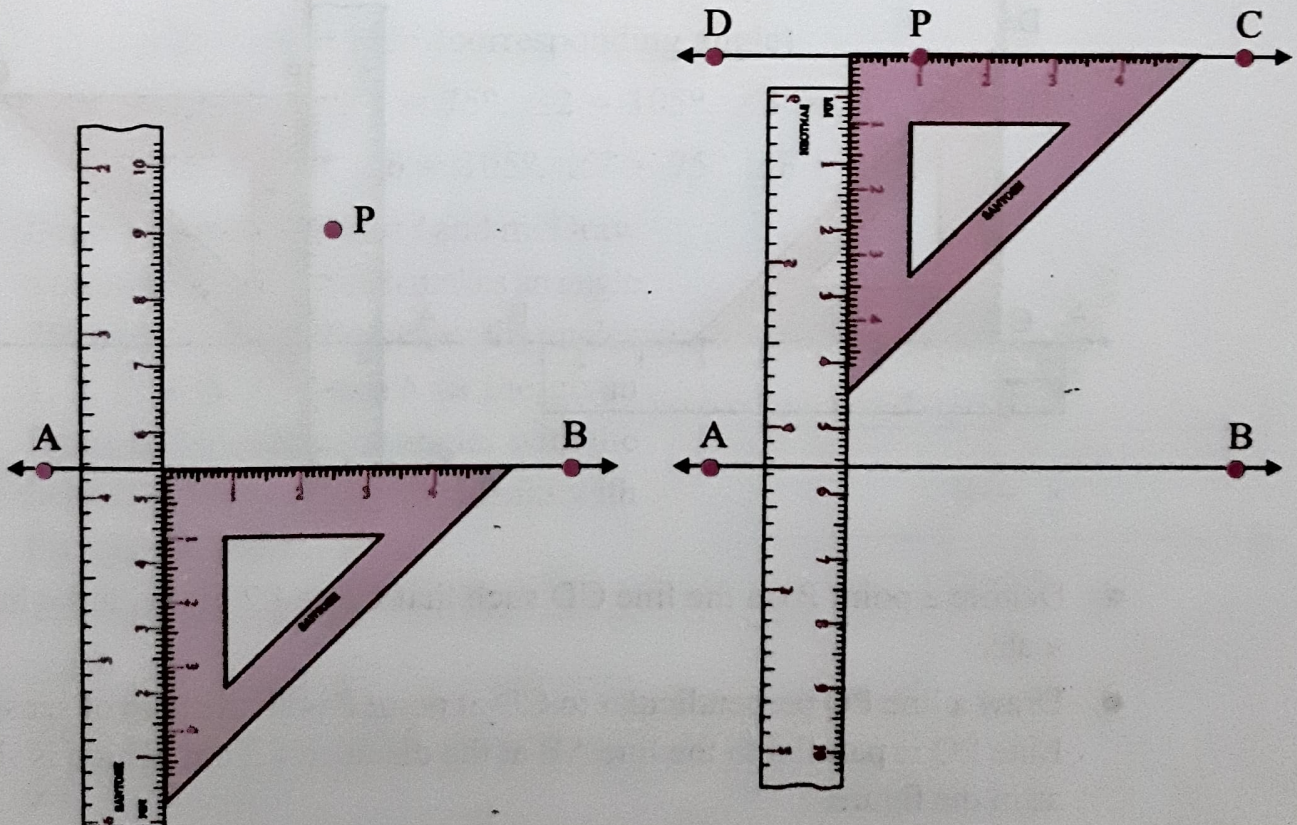


Draw a line parallel to a given line which passes through a given point with the help of scale and set square :

Example 1 : Draw a line parallel to the line AB which passes through a point P which is outside to this line.

Solution : Put the side of right angle of set square on the line AB as given figure.

- We hold the set square as it is stable at its position.
- Put a scale beside the second side of the set square.



- Slide the set square with the help of scale such that the side of right angle of set square touches the point P.
- Draw the line CD with the help of set square which passes through point P. This line CD is parallel to the line AB.

Think it

- How many lines can be drawn from the point P ?
- The following axiom of great mathematician Playfair in this reference :

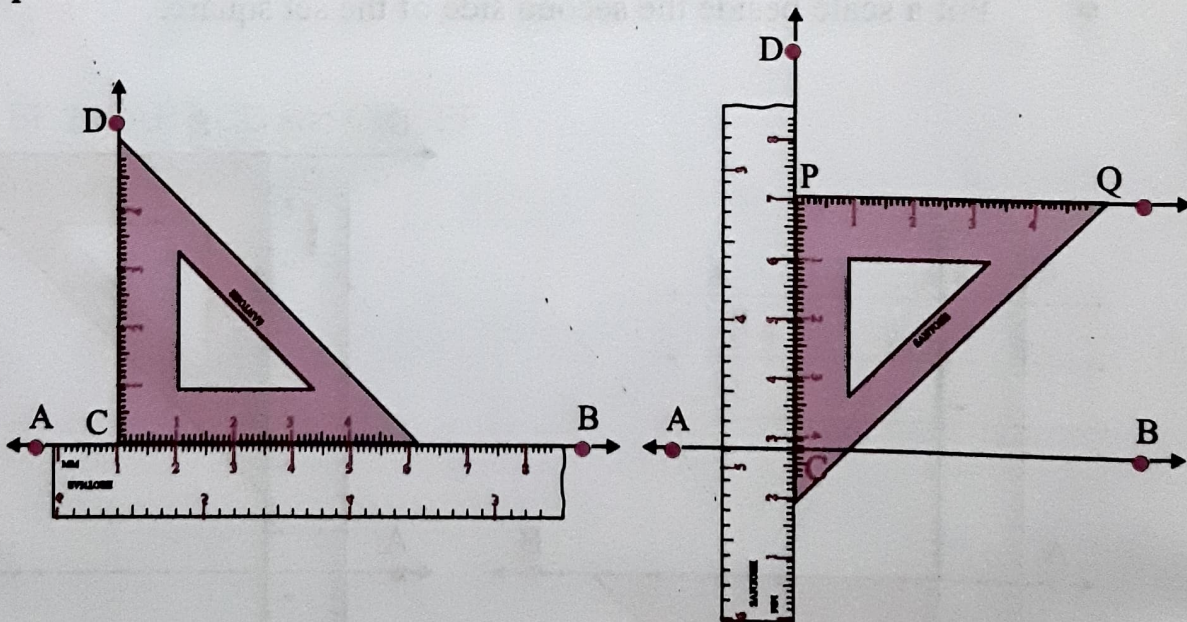
Playfair's axiom : One and only one parallel line can be drawn from a given point parallel to a given line, when that point is not on the line.

9.12 : Draw a parallel line with the help of scale and set square to given distance from the given line.

Example 2 : Draw a line parallel to the line AB which is at the distance of 4.2 cm from line AB.

Solution :

- Draw line AB. Denote a point C on it.
- Draw line CD perpendicular to the line AB at point C with the help of set square.



- Denote a point P on the line CD such that $CP = 4.2$ cm with the help of scale.
- Draw a line PQ perpendicular to CD at point P with the help of set square. Line PQ is parallel to the line AB at the distance 4.2 cm, which is shown as in the figure.

9.13 : Find the measurement of unknown angle with the help of characteristics of parallel lines and verify it to draw the lines.

Example 3 : In the following figure $l \parallel m$, transversal line n cuts it at the points P and Q. The angles formed by it have represented by 1, 2, 3, 4, 5, 6, 7 and 8.

If $\angle 1 = 75^\circ$ then find the rest of the angles.

Solution : Given that $\angle 1 = 75^\circ$
but $\angle 3 = \angle 1$ (vertically opposite)

$$\therefore \angle 3 = 75^\circ$$

Again, $\angle 3 = 75^\circ$ (alternate angle)

$$\therefore \angle 5 = 75^\circ$$

Now, $\angle 5 + \angle 4 = 180^\circ$ (interior angle of same side of transversal line)

$$\therefore \angle 4 = (180^\circ - \angle 5)$$

$$= 180^\circ - 75^\circ = 105^\circ$$

and, $\angle 3 + \angle 6 = 180^\circ$ (interior angle of same side of transversal line)

$$\therefore \angle 6 = (180^\circ - \angle 3) = (180^\circ - 75^\circ) = 105^\circ$$

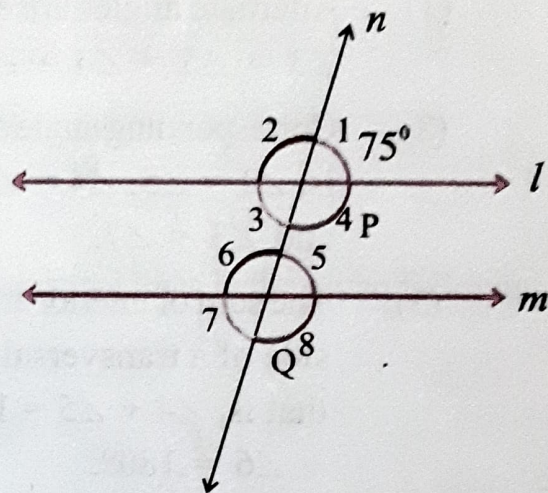
$$\angle 2 = \angle 6 = 105^\circ \text{ (corresponding angle)}$$

$$\therefore \angle 7 = \angle 3 = 75^\circ \text{ (corresponding angle)}$$

$$\angle 8 = \angle 4 = 105^\circ \text{ (corresponding angle)}$$

Therefore, $\angle 1 = 75^\circ$, $\angle 2 = 105^\circ$, $\angle 3 = 75^\circ$, $\angle 4 = 105^\circ$

$$\angle 5 = 75^\circ, \angle 6 = 105^\circ, \angle 7 = 75^\circ, \angle 8 = 105^\circ$$



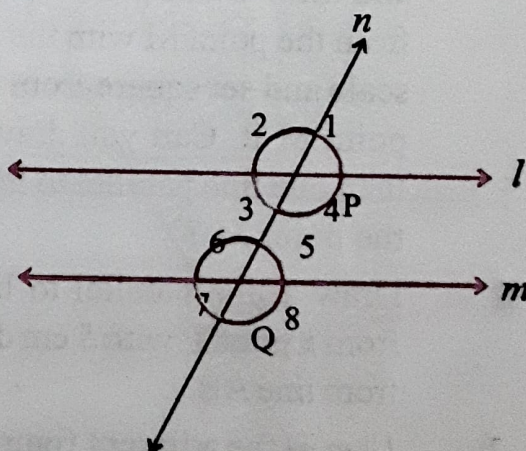
- Draw two parallel lines l and m . Draw a transversal line n which makes an angle 75° with the line l . Represent the angles 1, 2, 3, 4, 5, 6, 7 and 8 as the given figure and measure the angles with the help of protector. Fill in the blanks with the measurement :

$$\angle 2 = \dots\dots\dots, \angle 3 = \dots\dots\dots,$$

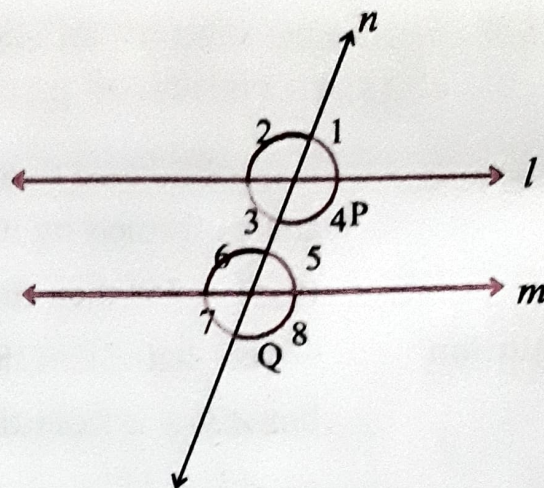
$$\angle 4 = \dots\dots\dots, \angle 5 = \dots\dots\dots$$

$$\angle 6 = \dots\dots\dots, \angle 7 = \dots\dots\dots,$$

$$\angle 8 = \dots\dots\dots$$



- There are two parallel lines l and m in the given figure. Draw transversal line n which cuts line l and m at P and Q respectively. Eight angles are formed by it



- Alternate angles are equal that is, $\angle 3 = \angle 5 = \angle 4 = \angle 6$
- Corresponding angles are equal that is, $\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 2 = \angle 6$ and $\angle 3 = \angle 7$.
- The sum of interior angles of same side of a transversal line is 180° that is, $\angle 4 + \angle 5 = 180^\circ$ and $\angle 3 + \angle 6 = 180^\circ$.

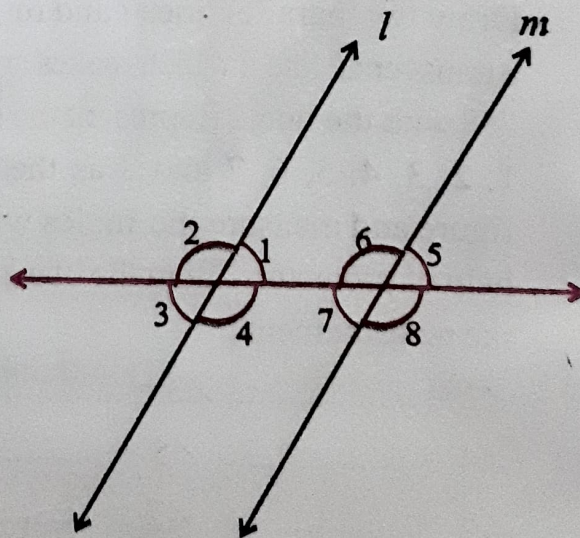
You draw the same type of figure in your note book, measure the angles and give confirmation of the above statements.

Therefore, we reach at these conclusion.

- If a transversal line cuts two parallel lines then the angles formed by them ;
 - Alternate angles are equal
 - Corresponding angles are equal.
 - The sum of interior angles of same side of a transversal line is 180° .

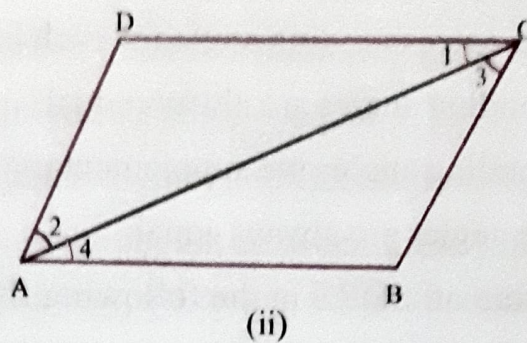
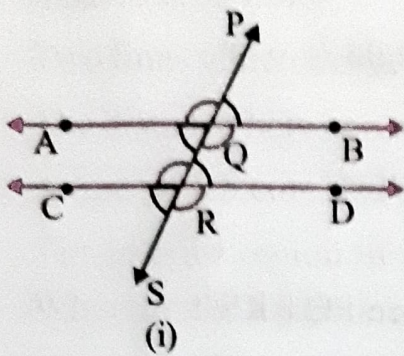
Also do it :

- Draw line PQ on your note book and draw a line parallel line PQ from the point M with the help of scale and set square from a outer point M it. Can you draw more than one line parallel to PQ from the point M ?
- Draw a line parallel to line AB from a point P with 5 cm distance from line AB .
- $l \parallel m$ in the adjacent figure and if $\angle 5 = 60^\circ$ then find the value of $\angle 2$, $\angle 3$, $\angle 4$, $\angle 6$, $\angle 7$ and $\angle 8$.

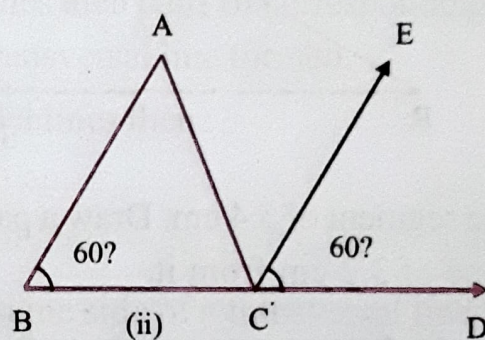
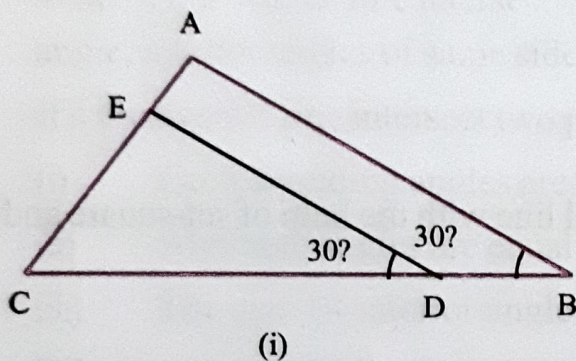


Efficiency Exercise 9

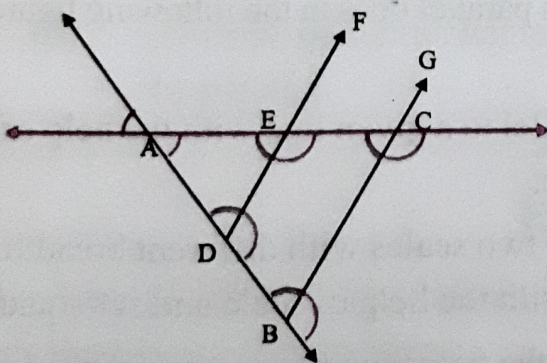
1. Write down the pair of alternate angles in the following figure.



2. Write down the name of pair of parallel lines and the transversal lines in the given following figure :

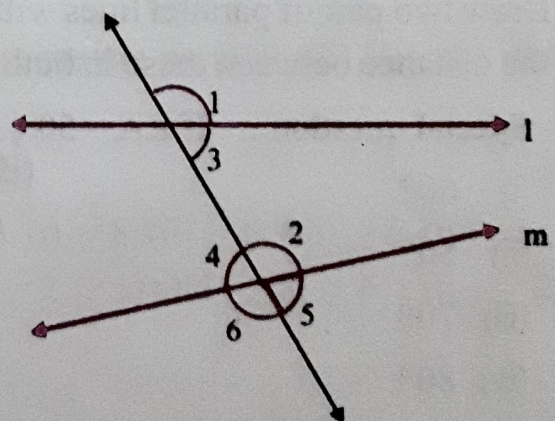


3. Describe the interior angles formed by transversal line on the parallel lines in the given following figure :

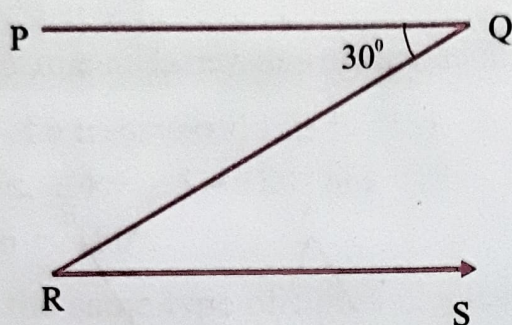


4. Find these from the following figure :

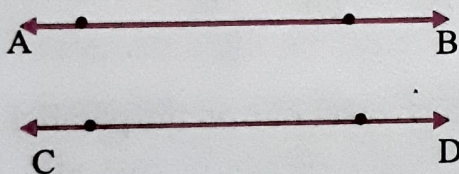
- (i) Which are the pairs of corresponding angles ?
- (ii) Which are the pair of interior angles of same side of transversal line ?
- (iii) Which are the pair of alternate angle ?



5. If a transversal line intersects two parallel lines then find the true statements from the following statements.
- Two pairs of alternate angle are formed ?
 - There is a corresponding angle for each interior angle ?
 - Corresponding angles are always equal.
 - Corresponding angles are supplementary to each other.
 - Alternate angles are always equal.
6. What is the value of $\angle QRS$ in the following figure, when $PQ \parallel RS$



7. Draw a line segment of 5.4 cm. Draw a parallel line with the help of set-square and scale at a distance of 3.2 cm from it.

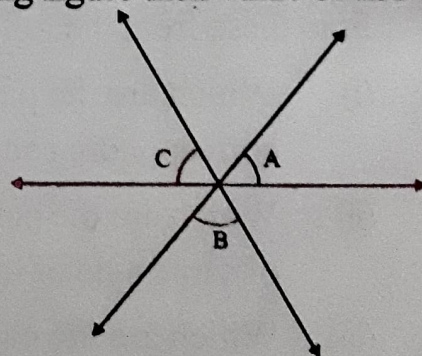


8. Find the perpendicular distance between the given parallel lines in the following figure with the help of set square and scale.
9. How to draw a line from an exterior point parallel to a given line with the help of scale and set-square ? Show it by drawing the figure.
10. Draw two pair of parallel lines with the help of two scales with different breadth. Find the distance between these in both condition with the help of scale and set-square.

Special question : If $\angle A = 50^\circ$, $\angle C = 60^\circ$ in the following figure then value of $\angle B$ is :

- 60°
- 50°
- 70°
- 80°

Answer (iii) 70°



We have learnt in this unit :

1. Those two intersecting lines for which angle between them is 90° , are called perpendicular lines to each other.
2. Two lines which never intersect each other in a plane, are called parallel lines.
3. The distance between two parallel lines is always equal.
4. A line which cuts two or more than two lines at different points is a transversal line.
5. The interior region of two lines is the region between these lines except that lines. Whereas, the exterior region is the rest part except these lines and its interior region.
6. There are eight angles formed if a transversal line intersect to two parallel lines, the angles of exterior region are exterior angles and the angles of interior region are interior angles.
7. When a transversal line intersect two lines then pairs of corresponding angle, alternate angle, interior angles of same side of transversal line formed.
8. If a transversal line intersect two parallel lines then :
 - (i) Corresponding angles are equal
 - (ii) Alternate angles are equal.
 - (iii) The sum of interior angles of same side of a transversal line is 180° .
9. When a transversal line intersects two lines and :
 - (i) Corresponding angles are equal
 - (ii) Alternate angles are equal.
 - (iii) The sum of interior angles of same side of transversal line is 180° then both lines be parallel.

ANSWERS

Exercise 9(a)

2. $CD = 3$ cm., 3. (i) Yes, The infront sides of reference are parallel, (ii) Yes, the distance between them is equal everywhere.

Exercise 9(b)

2. Interior point G, H and J, Exterior point I, K and L; 3. (i) $\angle 8$, (ii) $\angle 6$, (iii) $\angle 5$, (iv) $\angle 8$; 4. (i) $\angle 2$, $\angle 3$ and $\angle 6$, $\angle 7$ (ii) ($\angle 1$, $\angle 3$) ($\angle 7$, $\angle 4$), ($\angle 2$, $\angle 5$) and ($\angle 8$, $\angle 6$); (iii) $\angle 2$, $\angle 6$ and $\angle 7$, $\angle 3$ (iv) $\angle 1$, $\angle 8$ and $\angle 4$, $\angle 5$

Exercise 9 (c)

1. No, corresponding angle and alternate angle are not equal, 2. Yes, corresponding angle is equal, 3. Corresponding angles.

Efficiency Exercise 9

1. (i) $\angle BQR$, $\angle QRC$, and $\angle AQR$, $\angle QRD$; (ii) $\angle DCA$, $\angle CAB$ and $\angle DAC$, $\angle ACB$; 2. (i) $DE \parallel AB$, transversal line BC (ii) $AB \parallel EC$ transversal line BD ; 3. $AB \dots\dots \angle EDB$ and $\angle CBD$, $AC \dots\dots \angle DEC$ and $\angle ECB$ 4. (i) $\angle 1$, $\angle 2$ and $\angle 3$, $\angle 5$ (ii) $\angle 3$, $\angle 2$ (iii) $\angle 3$, $\angle 4$ 5. (i) Yes (ii) Yes (iii) Yes (iv) No (v) Yes 6. 30°

