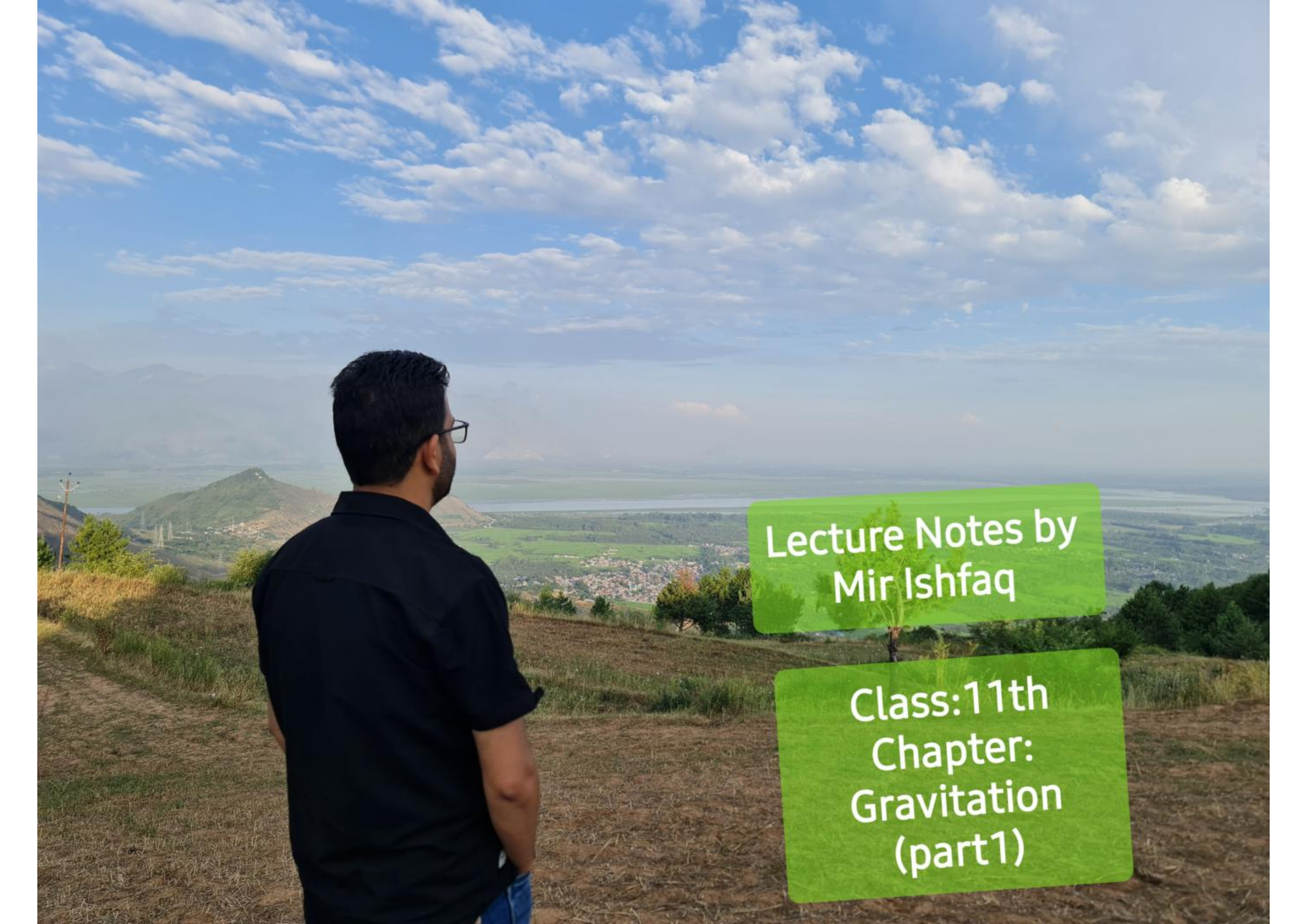




Lecture Notes by
Mir Ishfaq

Class:11th
Chapter:
Gravitation
(part1)

Gravitation:-

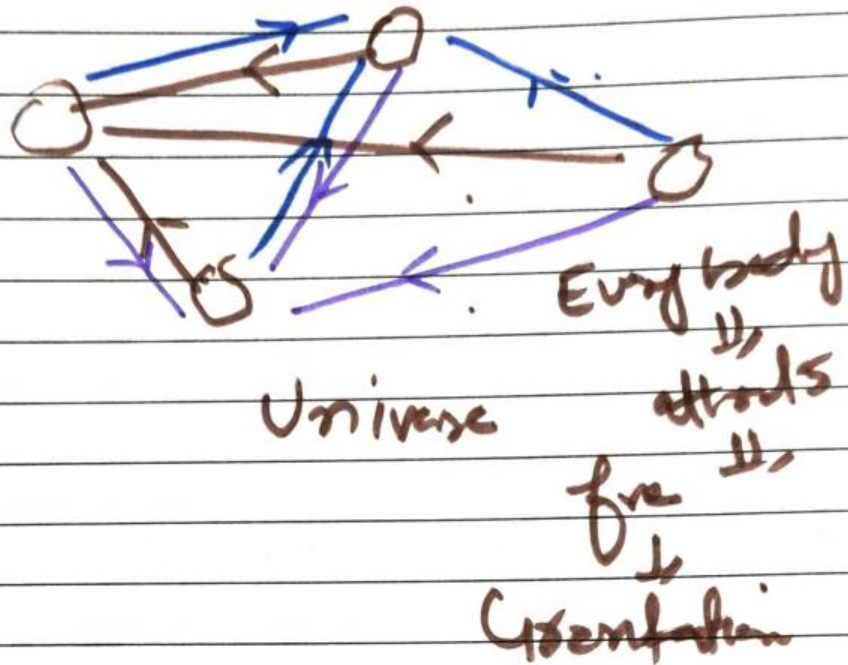
IP-1 11th

06/10/20

classmate

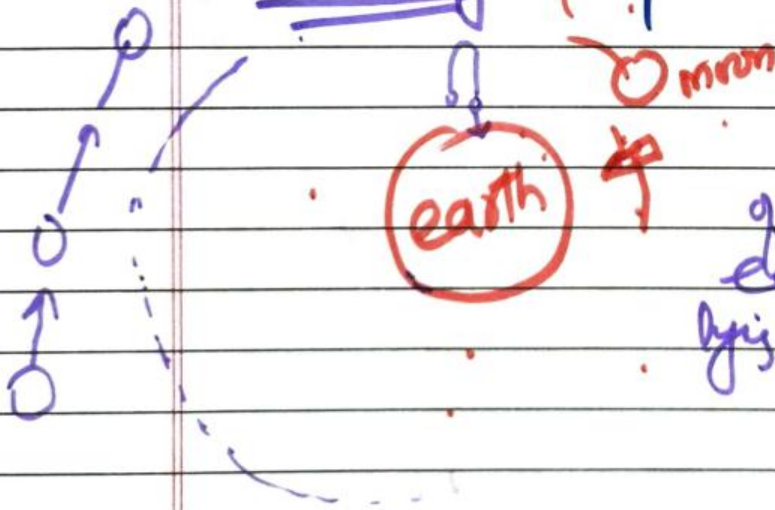
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Gravitation is the force of attraction b/w any two bodies in the universe.

Gravity:- Special case of gravitation



Gravity is the force of attraction b/w the earth and any object lying on or near its surface.

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Free fall:- The motion of a body under the influence of gravity alone is called free fall.
(Vacuum) ↓

Acceleration due to gravity:-

$$a = \frac{v_1 - u}{t} = \frac{9.8 - 0}{1}$$

A • $u = 0$ (rest)
B • $v_1 = 9.8 \text{ ms}^{-1}$

$$a = 9.8 \text{ ms}^{-2}$$

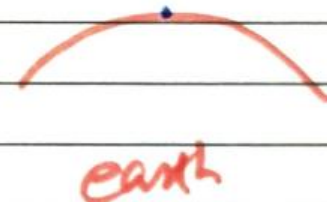
C • $v_2 = 19.6$

btw B & C.

$$a = \frac{v_2 - v_1}{t} = \frac{19.6 - 9.8}{1}$$

D • $v_3 =$ $\begin{matrix} +9.8 \\ \downarrow \end{matrix}$

$$a = 9.8 \text{ ms}^{-2}$$



btw C & D.

$$a = \frac{v_3 - v_2}{t} = 9.8 \text{ ms}^{-2}$$

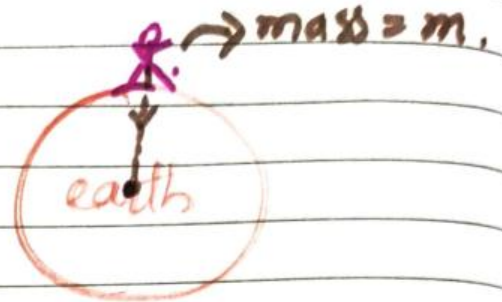
↳ acceleration due to gravity = g .

The acceleration produced in a freely falling body under the gravitational pull of the earth is called acceleration due to gravity.

$$g = 9.8 \text{ ms}^{-2} \text{ (const)} \rightarrow \begin{matrix} \text{near the} \\ \text{surface} \\ \text{at earth.} \end{matrix}$$

Weight of a body:-

It is defined as the gravitational force with which a body is attracted towards the center of earth.



$= ma$

$$W = mg$$

eg person mass 60 kg.
 $g = 10 \text{ m/s}^2$

$$\therefore \text{Weight of person} = W = mg \\ = 60 \times 10$$

$$W = 600 \text{ N}$$

Travel to moon. $\rightarrow g_{\text{moon}} = \frac{g_e}{6}$

person mass 60 kg.

weight on moon $= W_m = m g_{\text{moon}}$

$$W_m = 60 \times \frac{g_e}{6} \quad \text{on jupiter}$$

$$W_m = 10 g_e = 10 \times 10$$

$$W_m = 100 \text{ N}$$

$$g_j = 10 g_e$$

$$W_j = m g_j$$

$$= 60 \times 10 \times 10 \\ = 6000 \text{ N}$$

at center of earth, or in orbit or free fall
 $g = 0$

$$W = mg \\ = 60 \times 0$$

$$W = 0$$

$$P = 4 \times 10^{15}$$

on earth.

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10^{10} protons
 10^{11} neutrons



$\Rightarrow$ atoms

Protons Neutrons electron
fundamental particles

Total Mass

$$= 10^{10} \times m_p + 10^{11} \times m_n$$

$$M = 50 \text{ Kg} \quad (\text{Suppose})$$

Shift this person to Moon:

on

Moon

10^{10} protons
 10^{11} neutrons



$\Rightarrow$ atoms
P N e

Total Mass on moon.

$$= 10^{10} \times m_p + 10^{11} \times m_n$$

$$M_{\text{moon}} = 50 \text{ Kg}$$

Mass

weight

1. quantity of matter

force with which earth attracts u.

2. Mass is same, at different places

weight is diff at diff places (on different planets)

3. S.I unit mass = Kg

S.I unit weight = N

4. It is scalar qty

It is a vector quantity.

Newton's Law of Gravitation:

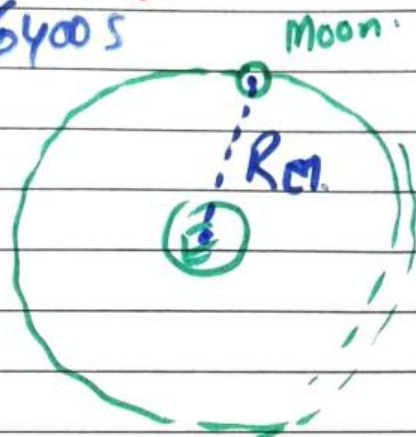
Moon.

$$T = 27.3 \text{ days}$$

$$R_M = 3.85 \times 10^5 \text{ km}$$

$$T = 27.3 \text{ days} = 27.3 \times 86400 \text{ s}$$

$$R_M = 3.85 \times 10^8 \text{ m}$$



Speed of moon

$$v = \frac{\text{Circumference of orbit}}{\text{Orbital period}} = \frac{2\pi R_M}{T}$$

$$v = \frac{2\pi \times (3.85 \times 10^8 \text{ m})}{27.3 \times 86400 \text{ s}}$$

$$v = 1.02 \times 10^3 \text{ ms}^{-1}$$

$$\omega = \frac{2\pi}{T}$$

The acceleration of moon.

$$a_m = \frac{v^2}{R} = \frac{(1.02 \times 10^3)^2}{3.85 \times 10^8}$$

$$a_m = \omega^2 R$$

$$a_m = \frac{(2\pi)^2 R}{T^2}$$

$$a_m = 2.72 \times 10^{-3} \text{ ms}^{-2}$$

$$a_m = \frac{4\pi^2 \times (3.85 \times 10^8 \text{ m})}{(27.3 \text{ days})^2}$$

$$= 0.0027 \text{ ms}^{-2}$$

$$= 2.72 \times 10^{-3} \text{ ms}^{-2}$$

Close to surface of earth

$$a_{apple} = g = 9.8 \text{ m s}^{-2} \quad \text{--- (ii)}$$

$$\therefore \frac{a_{apple}}{a_{moon}} = \frac{9.8}{2.72 \times 10^{-3}}$$



$$\frac{a_{apple}}{a_{moon}} = 3600 \quad \text{--- (iii)}$$

Newton assumed.

$$\frac{R_{moon}}{R_{apple}} = \frac{3.85 \times 10^8}{6.4 \times 10^6}$$

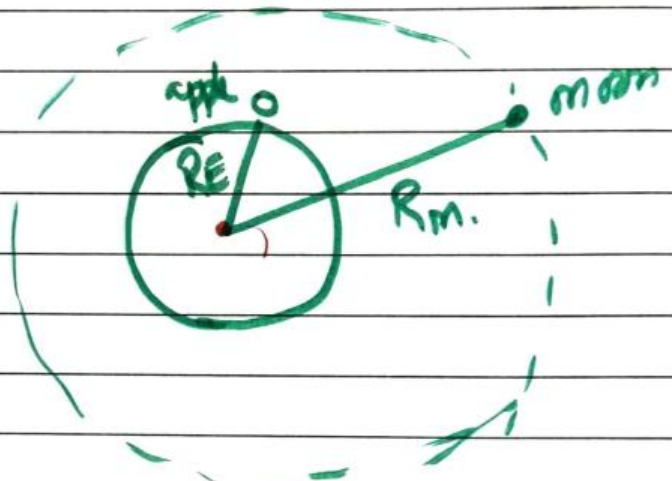
$$\frac{R_{moon}}{R_{apple}} = 60$$

Squaring both

R_{moon} = distance of moon from earth
 R_{apple} = " " apple from earth

$$\left(\frac{R_{moon}}{R_{apple}} \right)^2 = 3600 \quad \text{--- (iv)}$$

from (iii) & (iv)

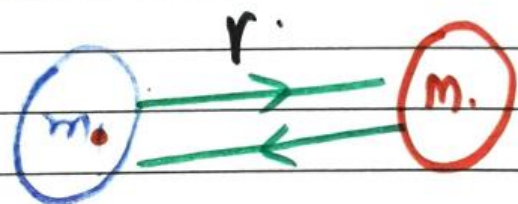


$$\frac{a_{\text{apple}}}{a_{\text{moon}}} = \left(\frac{R_{\text{moon}}}{R_{\text{apple}}} \right)^2$$

$$a \propto \frac{1}{r^2}$$

$$\therefore F \propto m \times \frac{1}{r^2}$$

Newton third law



$$F \propto \frac{mM}{r^2}$$

If $m \rightarrow 0$, expt. $F = 0$

$$\text{Sum } F \propto \frac{m+M}{r^2} \Rightarrow F \propto \frac{0+M}{r^2} \propto \frac{M}{r^2} = \frac{10^x \text{ N} \cdot \text{N}}{r^2} \text{ (Theoretical)}$$

$$\text{Diff } F \propto \frac{m-M}{r^2} \Rightarrow F \propto \frac{0-M}{r^2} \propto \frac{M}{r^2} = \frac{10^x \text{ N} \cdot \text{N}}{r^2} \text{ (Theoretical)}$$

$$\text{Div } F \propto \frac{M}{m r^2} \Rightarrow F \propto \frac{M}{0} \propto \infty = \infty \text{ N (Theoretical)}$$

$$\text{Product } F \propto \frac{mM}{r^2} \Rightarrow F \propto \frac{0M}{r^2} \Rightarrow \boxed{F=0}$$

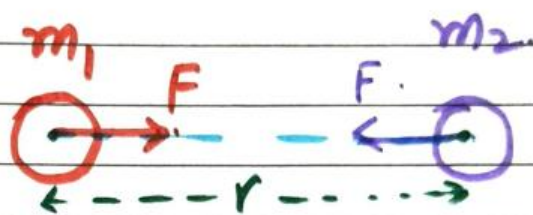
Practical Theoretical

Newton's Universal law of Gravitation:

Statement:- Every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance b/w them. This force acts along the line joining the two particles.

$$F \propto m_1 m_2$$

$$\text{and } F \propto \frac{1}{r^2}$$



Combining these

$$F \propto \frac{m_1 m_2}{r^2}$$

$$F = G \frac{m_1 m_2}{r^2}$$

where G is a constant & is known as Universal gravitational constant.

Def of G :- If $m_1 = m_2 = 1$, $r = 1$, then

$$F = G \frac{1 \times 1}{1^2} \Rightarrow \boxed{F = G} \text{ The universal gravitat}$$

constant may be defined as the force of attraction b/w two bodies of unit mass each and placed unit distance apart.

S.I Value of G is

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ Kg}^{-2}$$

C.g.s $\rightarrow$ Value of G is

$$G = 6.67 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$$

Units of G :

$$F = G \frac{m_1 m_2}{r^2}$$

$$G = \frac{F \times r^2}{m_1 m_2}$$

$$\text{S.I units of } G = \frac{\text{Nm}^2}{\text{Kg} \times \text{Kg}} = \text{Nm}^2 \text{ Kg}^{-2}$$

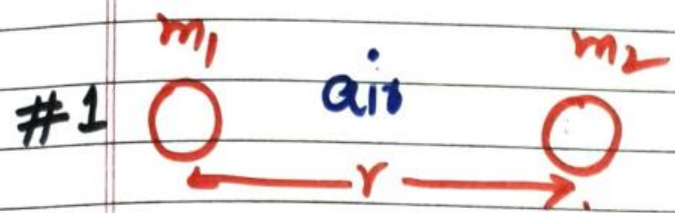
$$\text{C.g.s of } G = \frac{\text{dyne} \times \text{cm}^2}{\text{g} \times \text{g}} = \text{dyne cm}^2 \text{ g}^{-2}$$

Dimensional formula:

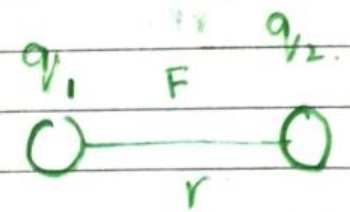
$$[G] = \frac{M^1 L^1 T^{-2} \times L^2}{M^1 M^1} = [M^{-1} L^3 T^{-2}]$$

Gravitation (masses)

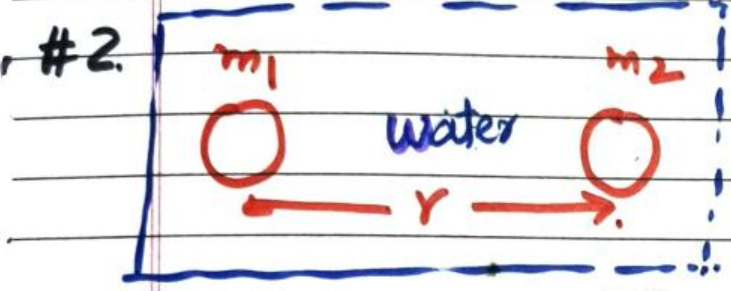
Charges



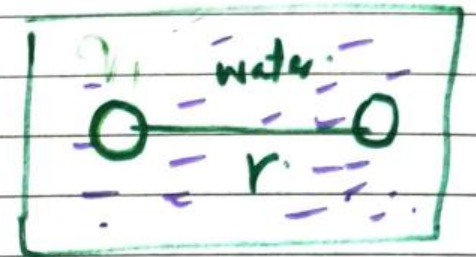
In air, $F = 5N$



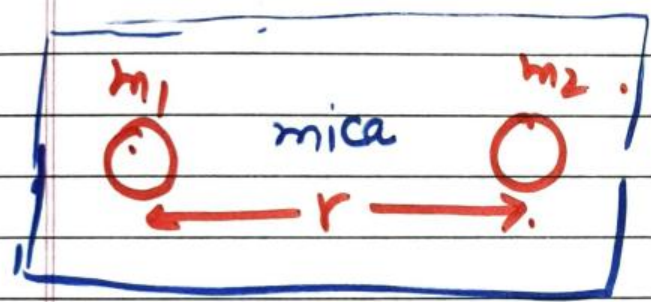
In air $F = 80N$



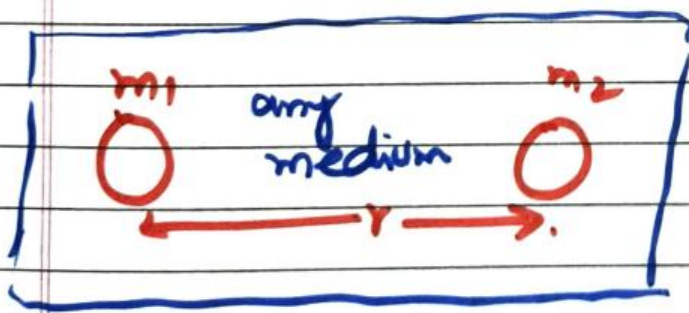
In water, $F = 5N$



In water, $F = 1N$



In mica, $F = 5N$



In any medium, $F = 5N$

The value of G does not depend on the nature & size of bodies.

It also does not depend on the nature of the medium b/w the two bodies.

i.e. why G is called Universal gravitational constant.

$$\frac{GMm}{r^2} L = k\theta$$

or

$$G = \frac{k\theta r^2}{MmL}$$

Knowing all the quantities on the right hand side from the experiment, the value of G can be determined.

Since Cavendish's experiment, the measurement of G has been improved upon. The currently accepted value is

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}.$$

8.4 EVIDENCES IN SUPPORT OF LAW OF GRAVITATION

8. Mention some experimental evidences in support of Newton's law of gravitation.

Experimental evidences in support of the law of gravitation. Many of the results predicted theoretically on the basis of the law of gravitation are found to be in close agreement with the experimental observations. Some of such evidences are as follows :

- The rotation of the earth around the sun or that of the moon around the earth is explained on the basis of this law.
- The tides are formed in oceans due to the gravitational force of attraction between the moon and sea-water.
- The times of solar and lunar eclipses calculated on the basis of the law of gravitation are found to be reasonably accurate.
- The orbits and periods of revolutions of artificial satellites can be predicted very accurately on the basis of this law.
- The value of g varies from place to place on the surface of the earth in accordance with the law of gravitation.

8.5 VECTOR FORM OF THE LAW OF GRAVITATION

9. Express law of gravitation in vector form. What are its implications ?

Vector form of Newton's Law of Gravitation. As shown in Fig. 8.4, consider two particles A and B of masses m_1 and m_2 and separated by distance r .

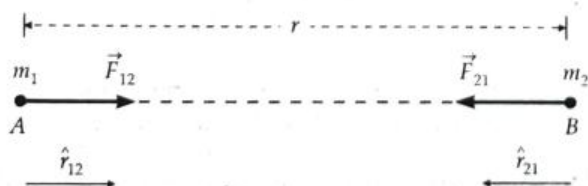


Fig. 8.4 Law of gravitation in vector form.

Let

$\hat{r}_{12}$ = a unit vector from A to B

$\hat{r}_{21}$ = a unit vector from B to A

$\vec{F}_{12}$ = gravitational force exerted on A by B

$\vec{F}_{21}$ = gravitational force exerted on B by A

In vector form, Newton's law of gravitation can be expressed as

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{21}$$

The negative sign shows that the direction of force $\vec{F}_{12}$ is opposite to that of $\hat{r}_{21}$ i.e., the gravitational force is attractive in nature so that m_1 is attracted towards m_2 .

$$\text{Similarly, } \vec{F}_{21} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

$$\text{But } \hat{r}_{21} = -\hat{r}_{12}$$

$$\text{Hence } \vec{F}_{21} = -\vec{F}_{12}$$

Thus the vector form of the law of gravitation implies that the gravitational forces acting between two particles form *action and reaction pair*. As $\vec{F}_{12}$ and $\vec{F}_{21}$ are directed towards the centres of the two particles, so gravitational force is a *central force*.

8.6 IMPORTANT FEATURES OF GRAVITATIONAL FORCE

10. Mention the characteristic features of gravitational force.

Important features of gravitational force :

- The gravitational force between two masses is independent of intervening medium.
- The mutual gravitational forces between two bodies are equal and opposite i.e., they form action and reaction pair. Hence *gravitational forces obey Newton's third law of motion*.
- The law of gravitation strictly holds for point masses.
- The gravitational force between two point masses is a central force. Its magnitude depends only on r and has no angular dependence. Thus the gravitational force possesses *spherical symmetry*.
- The gravitational force is a conservative force.
- The gravitational force between two bodies is independent of the presence of other bodies.

interaction particle. Graviton.

8.7 ▼ PRINCIPLE OF SUPERPOSITION OF GRAVITATIONAL FORCES

11. State and illustrate the principle of superposition of gravitational forces.

Principle of superposition of gravitational forces.

According to the principle of superposition, the gravitational force between two masses acts independently and uninfluenced by the presence of other bodies. Hence the resultant gravitational force acting on a particle due to a number of masses is the vector sum of the gravitational forces exerted by the individual masses on the given particle. Mathematically,

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots + \vec{F}_n = \sum_{i=1}^n \vec{F}_i$$

where $\vec{F}_1, \vec{F}_2, \vec{F}_3, \dots, \vec{F}_n$ are the gravitational forces exerted individually by n masses $m_1, m_2, m_3, \dots, m_n$ on the particle of mass m . Each force is determined by the law of gravitation.

Illustration. In Fig. 8.5, the total gravitational force on point mass m_1 is the vector sum of the gravitational forces exerted by the point masses m_2, m_3 and m_4 . According to the law of gravitation, the force on m_1 exerted by m_2 is

$$\vec{F}_{12} = -G \frac{m_2 m_1}{r_{21}^2} \hat{r}_{21}$$

where $\hat{r}_{21} = \vec{r}_{21} / r_{21}$ is a unit vector pointing from m_2 to m_1 . The negative sign shows that m_1 is being attracted by m_2 .

By the principle of superposition, the total gravitational force on m_1 is

Fig. 8.5 Principle of superposition of gravitational forces.

$$\begin{aligned} \vec{F}_1 &= \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} \\ &= -G \left[\frac{m_2 m_1}{r_{21}^2} \hat{r}_{21} + \frac{m_3 m_1}{r_{31}^2} \hat{r}_{31} + \frac{m_4 m_1}{r_{41}^2} \hat{r}_{41} \right] \end{aligned}$$

8.8 ▼ SHELL THEOREM

12. State Newton's shell theorem for the gravitational force.

Newton's shell theorem. This theorem gives gravitational force on a point mass due to a spherical shell or a solid sphere. It can be stated as follows :

(i) If a point mass lies outside a uniform spherical shell / sphere with a spherically symmetric internal mass distribution, the shell/sphere attracts the point mass as if the entire mass of the shell/sphere were concentrated at its centre.

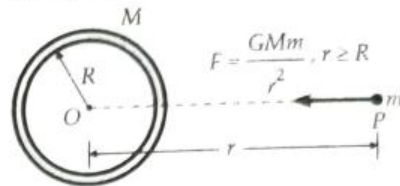


Fig. 8.6 Force on an outside particle due to a shell.

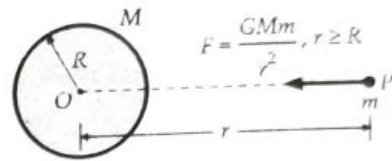


Fig. 8.7 Force on an outside particle due to a solid sphere.

Explanation. The gravitational forces exerted by the various regions of the shell/sphere have components along the line joining the point mass to the centre as well as along a direction perpendicular to this line. When summed over all the regions of the shell/sphere, the perpendicular components cancel out. Thus a resultant force acts along the joining the point mass to the centre.

(ii) If a point mass lies inside a uniform spherical shell, the gravitational force on the point mass is zero. But if a point mass lies inside a homogeneous solid sphere, the force on the point mass acts towards the centre of the sphere. This force is exerted by the spherical mass situated interior to the point mass.

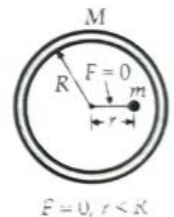
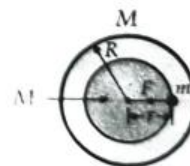


Fig. 8.8 No force acts on a particle inside a shell.



$$F = \frac{GM_r m}{r^2} = \frac{GMm}{R^3} r \left[\because \frac{M_r}{M} = \frac{r^3}{R^3} \right]$$

Fig. 8.9 A net force acts towards the centre on a particle inside ($r < R$) a solid sphere.

Explanation. Various regions of the spherical shell attract the point mass inside it in various directions. These forces cancel each other completely.

13. Is gravitational shielding possible ?

Gravitational shielding is not possible. Although the gravitational force on a particle inside a spherical shell is zero, yet the shell does not shield the other bodies outside it from exerting gravitational forces on the particle lying inside it. Thus gravitational shielding is not possible.



For Your Knowledge

- ▲ The law of gravitation is strictly true for point masses. However, it is valid in the following circumstances also :
 - ◆ For two bodies of any size provided they both have spherical symmetry.
 - ◆ When one body has spherical symmetry and the other is small compared with the separation between their centres.
 - ◆ When neither body has spherical symmetry but both are small compared with the separation between their centres.
- ▲ The value of G does not depend on the nature and size of the masses.
- ▲ The law of gravitation is universally valid. It applies to small objects on the earth, planets in the solar system and to galaxies.
- ▲ Though the gravitational force between two particles is central, the force between two finite rigid bodies is not necessarily along the line joining their centres of mass. For a particle lying outside a spherical symmetric body, the force acts as if the mass is concentrated at the centre of the body. Therefore, this force is central.
- ▲ The gravitational force on a particle lying inside a hollow body of any arbitrary shape is zero.

Examples based on

Newton's Law of Gravitation and Principle of Superposition

FORMULAE USED

1. Newton's law of gravitation, $F = \frac{G m_1 m_2}{r^2}$
2. Mass of planet or satellite, $M = \frac{4\pi^2 r^3}{GT^2}$
3. Principle of superposition of gravitational forces, $\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$

UNITS USED

Force of gravitation F is in newton, masses m_1, m_2 and M are in kg, distance r in metre and period of revolution in second.

CONSTANT USED

Universal gravitational constant,

$$G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}.$$

EXAMPLE 1. Calculate the force of attraction between two balls each of mass 1 kg each, when their centres are 10 cm apart. Given $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$. [Delhi 1997]

Solution. Here $m_1 = m_2 = 1 \text{ kg}$, $r = 10 \text{ cm} = 0.10 \text{ m}$

$$\begin{aligned} \therefore F &= \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 1 \times 1}{(0.10)^2} \\ &= 6.67 \times 10^{-9} \text{ N.} \end{aligned}$$

EXAMPLE 2. The mass of planet Jupiter is $1.9 \times 10^{27} \text{ kg}$ and that of the sun is $1.99 \times 10^{30} \text{ kg}$. The mean distance of the Jupiter from the Sun is $7.8 \times 10^{11} \text{ m}$. Calculate the gravitational force which the sun exerts on Jupiter. Assuming that Jupiter moves in a circular orbit around the sun, calculate the speed of the Jupiter.

Solution. Here $M = 1.99 \times 10^{30} \text{ kg}$,

$$m = 1.9 \times 10^{27} \text{ kg}, \quad r = 7.8 \times 10^{11} \text{ m},$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$$

$$\begin{aligned} \therefore F &= \frac{GMm}{r^2} = \frac{6.67 \times 10^{-11} \times 1.99 \times 10^{30} \times 1.9 \times 10^{27}}{(7.8 \times 10^{11})^2} \\ &= 4.145 \times 10^{23} \text{ N.} \end{aligned}$$

The gravitational force of attraction due to the sun provides the necessary centripetal force to the Jupiter to move in the circular orbit. If v is the orbital speed of Jupiter, then

$$\frac{mv^2}{r} = F$$

$$\begin{aligned} \text{or} \quad v &= \sqrt{\frac{rF}{m}} = \sqrt{\frac{7.8 \times 10^{11} \times 4.145 \times 10^{23}}{1.9 \times 10^{27}}} \\ &= 1.304 \times 10^4 \text{ ms}^{-1}. \end{aligned}$$

EXAMPLE 3. Two particles, each of mass m , go round a circle of radius R under the action of their mutual gravitational attraction. Find the speed of each particle.

Solution. The force on each particle is directed along the radius of the circle. The two particles will always lie at the ends of a diameter so that distance between them is $2R$.

$$\therefore F = G \frac{m \times m}{(2R)^2} = \frac{Gm^2}{4R^2}$$

As this force provides the centripetal force, so

$$\frac{Gm^2}{4R^2} = \frac{mv^2}{R}$$

$$\text{or} \quad v = \sqrt{\frac{Gm}{4R}}.$$

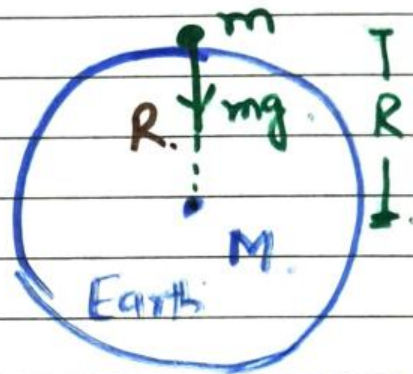
EXAMPLE 4. The mean orbital radius of the earth around the sun is $1.5 \times 10^8 \text{ km}$. Calculate the mass of the sun if $G = 6.67 \times 10^{-11} \text{ Nm}^2\text{kg}^{-2}$.

Relation b/w g and G :-

(Expression for acceleration due to gravity g)

Consider Earth to be sphere of mass M & radius R . Suppose a body of mass m is lying on its surface, then the force of attraction b/w the earth and the body.

$$F = G \frac{Mm}{R^2} \quad \text{--- (I)}$$



We use Shell theorem, entire mass at center of sphere. The force of gravity F an acceleration g in the body of mass m . From N & L of motion we get.

N & L $F = mg \quad \text{--- (II)}$

From eqn (I) & (II)

$$mg = G \frac{Mm}{R^2}$$

$$\boxed{g = \frac{GM}{R^2}} \quad \text{--- (III)}$$

The value of g is independent of the mass, size and shape of the body falling, under gravity (in Vacuum).

Mass of Earth:

$$g = \frac{GM}{R^2}$$

Mass of earth, $M = \frac{gR^2}{G}$

$$g = 9.8 \text{ ms}^{-2}, R = 6.37 \times 10^6 \text{ m}$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ Kg}^{-2}$$

$$\therefore M = \frac{9.8 \times (6.37 \times 10^6)^2}{6.67 \times 10^{-11}}$$

$$M = 5.97 \times 10^{24} \text{ Kg}$$

$$M = 6 \times 10^{24} \text{ Kg}$$

$$M_{\text{earth}} = 6 \times 10^{24} \text{ Kg}$$

Radius of earth

$$R^2 = \frac{GM}{g}$$

$$R = \sqrt{\frac{GM}{g}}$$

$$R = \sqrt{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{9.8}}$$

Acc due to gravity $g = \frac{GM}{R^2}$ $R = 6.37 \times 10^6 \text{ m}$

$$g = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24}}{(6.4 \times 10^6)^2} \approx 9.8 \text{ ms}^{-2}$$

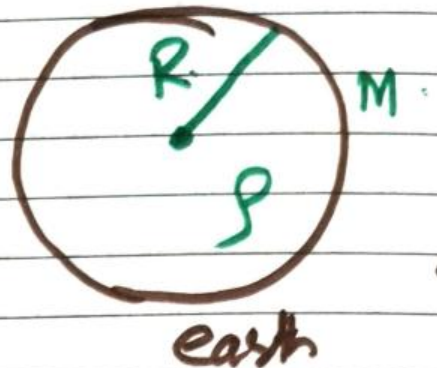
$$g = 9.8 \text{ ms}^{-2}$$

Average Density of earth:-

$$\rho = \frac{M}{V}$$

$$M = V \times \rho$$

$$M = \frac{4}{3} \pi R^3 \rho \quad \text{--- (I)}$$



Also $g = \frac{GM}{R^2}$

$$M = \frac{gR^2}{G} \quad \text{--- (II)}$$

From eq (I) & (II)

$$\frac{4}{3} \pi R^3 \rho = \frac{gR^2}{G} \Rightarrow \rho = \frac{g}{\frac{4}{3} \pi R G}$$

$$\rho = \frac{3g}{4\pi R G}$$

Knowing the value of g , R & G .

$$\rho = \frac{3 \times 9.8}{4 \times 3.14 \times 6.37 \times 10^6 \times 6.67 \times 10^{-11}} = 5497 \text{ kg m}^{-3}$$

$$\rho_{\text{earth}} = 5500 \text{ kg m}^{-3}$$

Variation of 'g' with altitude (height)

Consider the earth to be a sphere of mass M , radius R and center O .

Then the acceleration due to gravity at a pt A on the surface of earth will be.

$$g = \frac{GM}{R^2} \quad \text{--- (I)}$$



If g_h is the acceleration due to gravity at a point B at a height ' h ' from the earth's surface, then

$$g_h = \frac{GM}{(R+h)^2} \quad \text{--- (II)}$$

$\therefore$ Dividing (II) by (I)

$$\frac{g_h}{g} = \frac{GM}{(R+h)^2} \times \frac{R^2}{GM} = \frac{R^2}{(R+h)^2} \quad \text{--- (III)}$$

$$\frac{g_h}{g} = \frac{R^2}{R^2 \left[1 + \frac{h}{R}\right]^2} = \frac{1}{\left(1 + \frac{h}{R}\right)^2} \quad \text{--- (IV)}$$

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B.T

$$\frac{g_h}{g} = \left(1 + \frac{h}{R}\right)^{-2}$$

$$\text{if } x < 1$$

$$(1+x)^n = 1 + nx$$

if $h \ll R$, then $\frac{h}{R} \ll 1$,

∴ Using Binomial theorem and neglecting the higher powers of $\frac{h}{R}$.

$$\frac{g_h}{g} = \left(1 - \frac{2h}{R}\right)$$

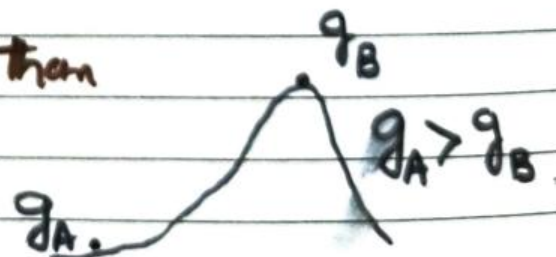
$$g_h = g \left(1 - \frac{2h}{R}\right) \quad \text{--- (W)}$$

$$\# \quad g = 9.8 \text{ ms}^{-2}, \quad \frac{2h}{R} = 0.5, \quad g_h = 9.8(1 - 0.5) \\ = 9.8(0.5)$$

$$g_h = 4.90 \text{ ms}^{-2}$$

∴ For ex (iii) & (iv), the value of acceleration due to gravity decreases with the increase of height h .

∴ g is less at mountains than at plains.



$$\text{Surface} = g, \text{ height } g_h = g\left(1 - \frac{2h}{R}\right)$$

#1. The decrease in value of g at height h .

$$\begin{aligned} g - g_h &= g - g\left(1 - \frac{2h}{R}\right) \\ &= g - g + \frac{2gh}{R} \end{aligned}$$

$$g - g_h = \frac{2gh}{R}$$

$$\text{clearly } g - g_h \propto h$$

#2. The percentage decrease in value of g at height h .

$$\frac{g - g_h}{g} \times 100 = \frac{2gh}{gR} \times 100\%$$

$$= \frac{2h}{R} \times 100\%$$

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#3 Width height the weight of the body decreases.

the loss in weight of a body at height h .

$$= mg - mg_h$$

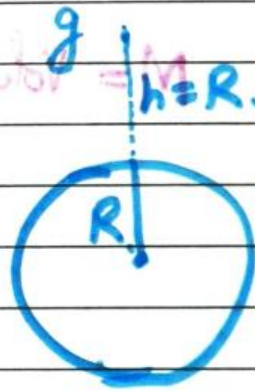
$$= m(g - g_h)$$

$$\text{loss in weight} = m \times \frac{2gh}{R}$$

#4. if $h = R$, then

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$g_h = g \left[\frac{R^2}{(R+h)^2} \right] \quad \text{--- (v)}$$



if $h = R$ (height = Radius) each.

$$g_{h=R} = g \left[\frac{R^2}{(R+R)^2} \right] = g \left[\frac{R^2}{(2R)^2} \right]$$

$$g_{h=R} = g \left[\frac{R^2}{4R^2} \right] \therefore g_{h=R} = \frac{g}{4}$$

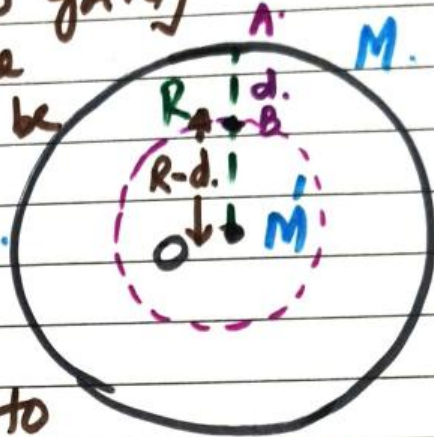
$$= \frac{9.8}{4} = 2.45$$

Variation of acceleration due to gravity 'g' with depth:-

Consider the earth to be a sphere of mass M , radius R and center O .

The acceleration due to gravity at any pt A on the surface of earth will be

$$g = \frac{GM}{R^2} \quad \text{--- (1)}$$



Assuming the earth to be a homogeneous sphere of average density ρ , then its total mass will be.

$$M = \text{Volume} \times \text{density}.$$

$$M = \frac{4}{3} \pi R^3 \times \rho$$

Using this in eqn (1)

$$g = \frac{G \times \frac{4}{3} \pi R^3 \rho}{R^2} =$$

$$g = \frac{4}{3} \pi G R \rho \quad \text{--- (II)}$$

Let g_d be the acceleration due to gravity at a point B at depth d below the surface of earth. A body at B is situated at the surface of inner solid sphere and lies inside the spherical

Shell of thickness d . The gravitational force of attraction on a body inside a spherical shell is always zero. Therefore, a body at B experiences gravitational force due to inner shaded sphere of radius $(R-d)$ & mass M' where.

$$M' = \text{Volume} \times \text{density} = \frac{4}{3} \pi (R-d)^3 \rho$$

$$\therefore g_d = \frac{GM'}{(R-d)^2} = \frac{G \frac{4}{3} \pi (R-d)^3 \rho}{(R-d)^2}$$

$$g_d = \frac{4}{3} \pi G (R-d) \rho \quad \text{--- (iii)}$$

$$\div \text{eqn (3) by eqn (ii)}$$

$$\frac{g_d}{g} = \frac{\cancel{\frac{4}{3}} \cancel{\pi} \cancel{G} (R-d) \rho}{\cancel{\frac{4}{3}} \cancel{\pi} \cancel{G} R \rho} = \frac{R-d}{R} = \frac{R}{R} - \frac{d}{R}$$

$$\frac{g_d}{g} = 1 - \frac{d}{R} \Rightarrow \boxed{g_d = g \left(1 - \frac{d}{R}\right)}$$

Clearly, the acceleration due to gravity decreases with the increase in depth d .

g is less in mines than that on earth's surface.

weight of a body at the centre of earth

$$g_d = g \left(1 - \frac{d}{R}\right)$$

$$= g \left(1 - \frac{R}{R}\right)$$

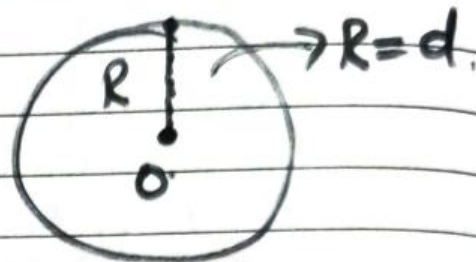
$$= g(1-1) = g(0) = 0$$

$$g_d = 0 \text{ [at the centre of earth]}$$

∴ weight of body of mass m at the centre of earth

$$W = mg_d = m \times 0 = 0$$

$$\boxed{W = 0}$$



Hence the weight of a body at the centre of earth is zero though its mass is not zero.

Decrease in value of g at depth d .

at surface of , at depth d , $g_d = g \left(1 - \frac{d}{R}\right)$

$$\text{Decrease } g - g_d = g - g \left(1 - \frac{d}{R}\right)$$

$$= \cancel{g} - \cancel{g} + \frac{gd}{R}$$

∴ Decrease in value of g at depth d is

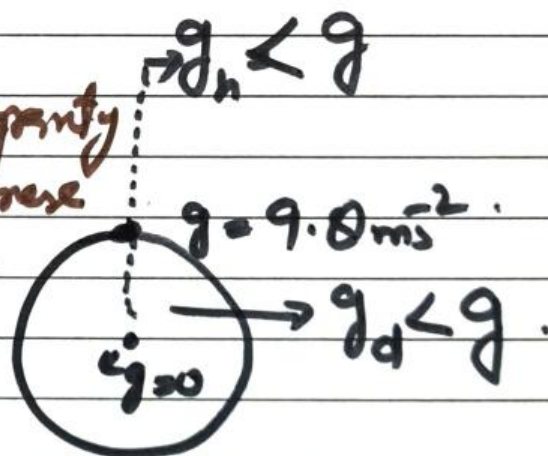
$$g - g_d = \frac{d}{R} g$$

Percentage decrease in the value of g at depth d is

$$\frac{g - g_d}{g} \times 100 = \frac{d}{R} \times 100\%$$

$$\% \text{ decrease} = \frac{d}{R} \times 100\%$$

The acceleration due to gravity decreases both with the increase in height and increase in depth. So it is max at the surface of earth and zero at the center of earth.



Weight of body decreases with height as well as depth & is maximum on the surface of earth. & zero at centre of earth.

Relation b/w height h & depth d for same change in g

At height h

$$g_h = g \left(1 - \frac{2h}{R} \right)$$

At depth d .

$$g_d = g \left(1 - \frac{d}{R} \right)$$

For the same chge in g ,

$$g_h = g_d$$

$$g \left(1 - \frac{2h}{R} \right) = g \left(1 - \frac{d}{R} \right)$$

$$\frac{2h}{R} = \frac{d}{R} \Rightarrow \boxed{d = 2h}$$

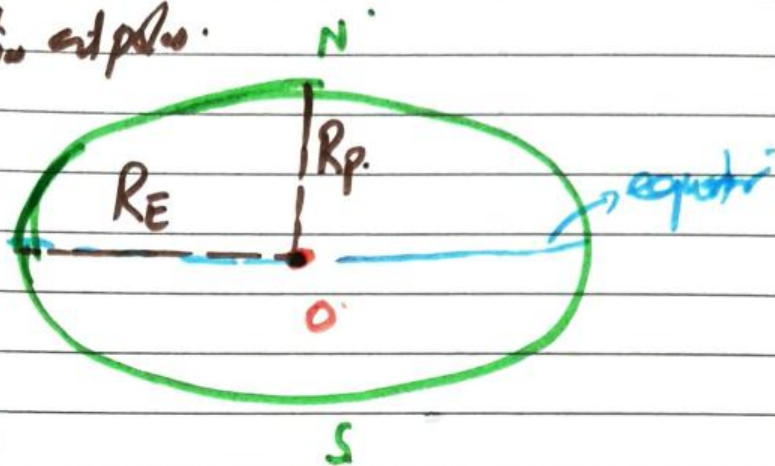
Hence acceleration due to gravity at a height h above the earth's surface will be same as that at depth $d = 2h$ below the earth's surface. But this fact holds only when $h \ll R$.

Variation of acceleration due to gravity with the shape of earth:-

Radius at eqr $R_E >$ Radius at pole.

$$R_E > R_P$$

$$R_E = R_P + 21 \text{ Km.}$$



Acc due to gravity

$$g = \frac{GM}{R^2}$$

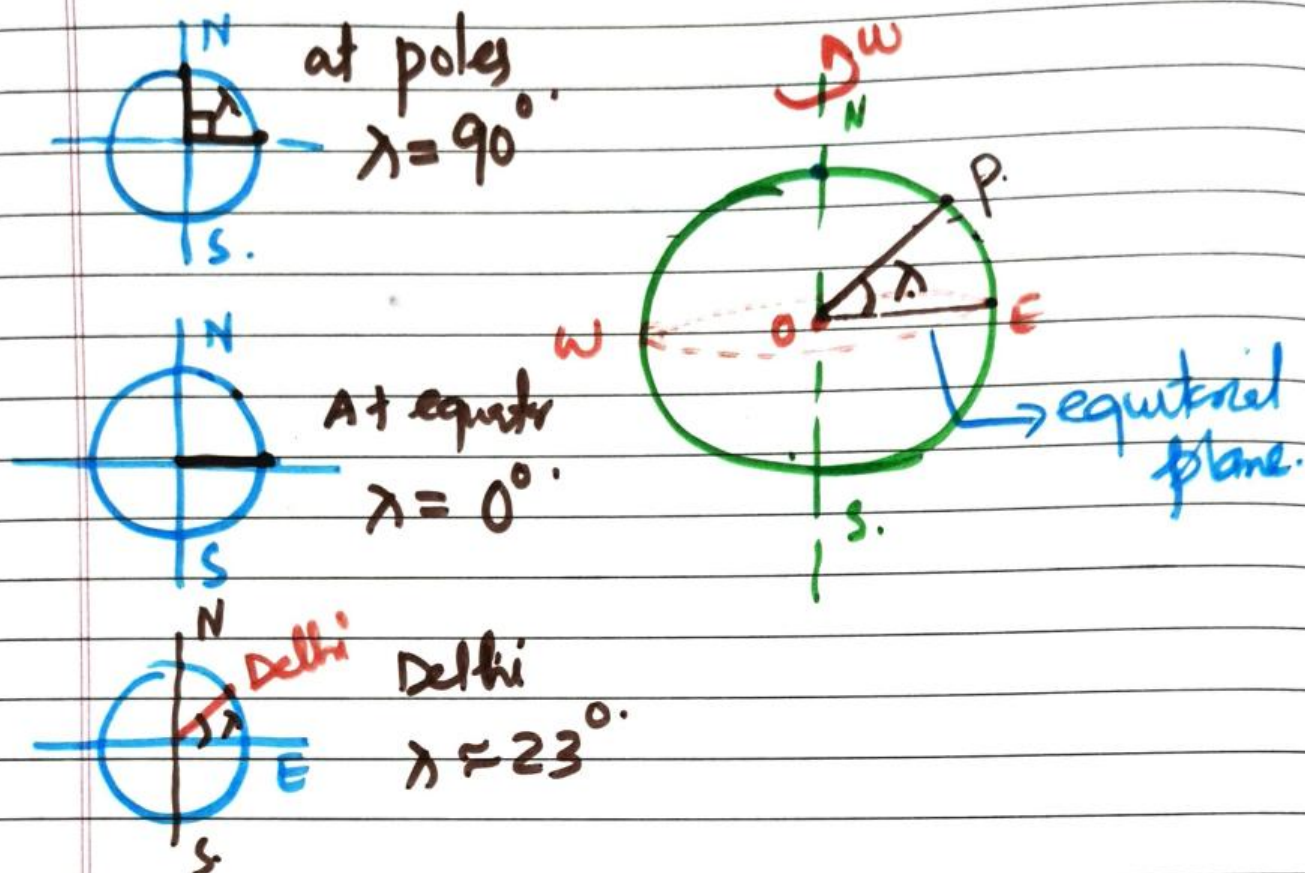
$$\therefore g \propto \frac{1}{R^2} \quad [\because G, M \text{ are const}]$$

$$\text{As } R_E > R_P$$

$$\Rightarrow g_E < g_P$$

Thus value of g is minimum at the equator and maximum at the poles. That is why the weight of the body increases when it is taken from the equator to the pole. The variation of g b/w the poles and the equator is about 0.5% .

Variation of g with Latitude:- (Effect on g due to rotation of earth).



Latitude of place P = $\angle POE = \lambda$..

Effect on g due to rotation of earth is given

$$g_{\lambda} = g - R\omega^2 \cos^2 \lambda$$

As λ increases, $\cos \lambda$ decreases and g_{λ} increases. So as we move from equator towards pole, the acceleration due to gravity increases.

Acc to gravity increases with the increase in Latitude of the place.

Special Cases:

1. At the equator, $\lambda = 0^\circ$

$$g_\lambda = g - R\omega^2 \cos^2 0^\circ$$

$$g_\lambda = g - R\omega^2$$

$$\therefore \boxed{g_E = g - R\omega^2}$$

Acceleration due to gravity decreases due to rotation of earth.

2. At the poles, $\lambda = 90^\circ$

$$g_\lambda = g - R\omega^2 \cos^2 90^\circ$$

$$g_\lambda = g$$

$$\therefore \boxed{g_P = g}$$

rotational motion of earth has no effect on the value of g at the poles.

The acceleration due to gravity is minimum at equator and maximum at poles. The difference in two values is

$$g_P - g_E = g - (g - R\omega^2) = R\omega^2$$

If the earth slows down or stops, the weight of the body will increase. (except at poles)

If earth rotates 17 times faster than present speed, the apparent weight of an object at equator becomes zero.

Variation of g :

1) For pts lying outside the earth ($r > R$)

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2} = \frac{R^2}{r^2} \quad \text{where } r = R+h.$$

$$g_h = \frac{gR^2}{r^2} \Rightarrow \boxed{g_h \propto \frac{1}{r^2}}$$

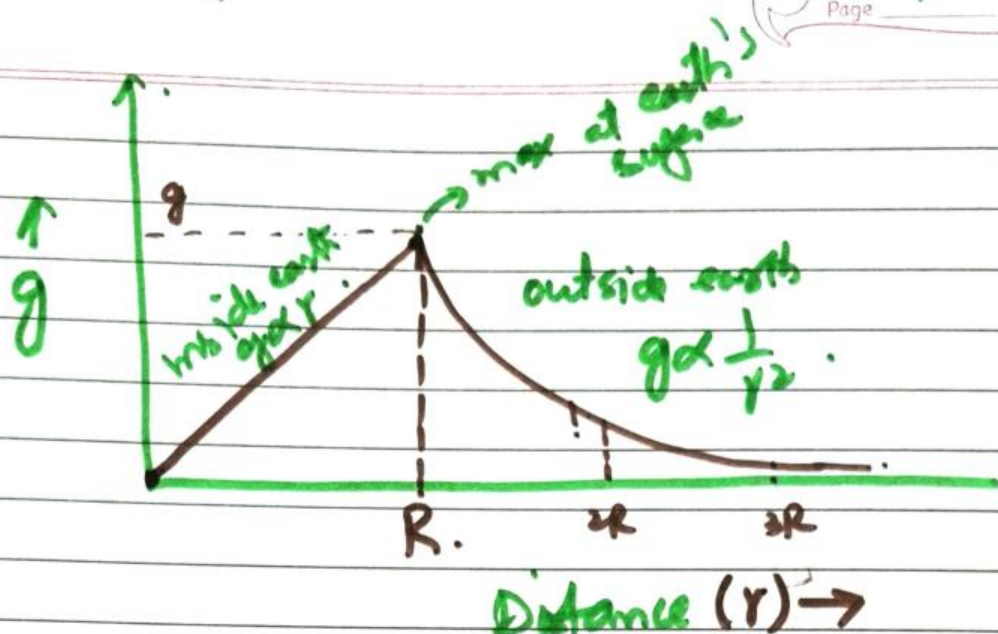
For pts lying inside the earth ($r < R$)

$$\frac{g_d}{g} = \frac{(R-d)}{R} = \frac{r}{R} \quad [\text{where } r = R-d]$$

$$g_d = \frac{gr}{R} \Rightarrow \boxed{g_d \propto r} \quad \left| \begin{array}{l} g_{\text{max}} \\ r = R \end{array} \right.$$

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Q How many times faster than its present speed the earth should rotate so that the apparent weight of an object at equator becomes zero. What would be the duration of the day in that case?

SA At the equator we have $mg_a = mg - mR\omega_1^2$.
As apparent weight at equator is zero,

$$mg_a = 0 \Rightarrow mg - mR\omega_1^2 = 0$$

$$\text{or } g = R\omega_1^2$$

$$\omega_1 = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{6.37 \times 10^6}} = 1.241 \times 10^{-3} \text{ rad s}^{-1}$$

But the present speed of earth

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.14}{24 \times 60 \times 60} = 7.27 \times 10^{-5} \text{ rad s}^{-1}$$

$$\therefore \frac{\omega_1}{\omega} = \frac{1.241 \times 10^{-3}}{7.27 \times 10^{-5}} = 17.06 \Rightarrow \frac{\omega_1}{\omega} = 17$$

i.e. earth should rotate 17 times faster than its present speed

$$\boxed{\omega_1 = 17\omega}$$

$$\text{New duration of day} = \frac{24}{17} = \underline{\underline{1.412 \text{ h}}}$$