

A man with dark hair and glasses, wearing a black hoodie, is holding a professional camera with a large lens. He is standing in a forest with green trees in the background. The image is used as a background for the text overlay.

Lecture Notes By Mir Ishfaq

Class:11th
Chapter:
Dimensional Analysis

Examples based on Measurement of Time

FORMULAE USED

$$\text{Fractional error in time} = \frac{\text{Difference in time}}{\text{Time interval}} = \frac{\Delta t}{t}$$

UNITS USED

Times t and Δt are in second.

EXAMPLE 25. The average life of an Indian is 56 years. Find the number of times the human heart beats in the life of an Indian, if the heart beats once in 0.8 s.

Solution. Average life of an Indian = 56 years
 $= 56 \times 365.25 \times 24 \times 60 \times 60 \text{ s}$

Period of heart beat = 0.8 s

Total number of heart beats in 56 years
 $= \frac{56 \times 365.25 \times 24 \times 60 \times 60}{0.8} = 2.2 \times 10^9 \text{ times.}$

EXAMPLE 26. The mean life of an elementary particle pion is 2×10^{-7} nanosecond. The age of the universe is about 4×10^9 years. Identify a physically meaning time interval that is approximately half way between these two on a logarithmic scale.

Solution. Mean life of pion,

$$t_1 = 2 \times 10^{-7} \text{ ns} = 2 \times 10^{-7} \times 10^{-9} \text{ s} \\ = 2 \times 10^{-16} \text{ s} \approx 10^{-16} \text{ s}$$

Mean life of universe,

$$t_2 = 4 \times 10^9 \text{ years} = 4 \times 10^9 \times 3 \times 10^7 \text{ s} \\ = 1.2 \times 10^{17} \text{ s} \approx 10^{17} \text{ s}$$

Let t be the time interval that is half way between t_1 and t_2 on the logarithmic scale. Then

$$\log t = \frac{1}{2} [\log t_1 + \log t_2] \\ = \frac{1}{2} [\log 10^{-16} + \log 10^{17}] \\ = \frac{1}{2} [-16 + 17] = 0.5 \approx 1$$

$$\therefore t = 10 \text{ s}$$

which is the time taken by an athlete to run a 100 m track.

PROBLEMS FOR PRACTICE

- Find the number of seconds in 1 year. Express them in order of magnitude. (Ans. $3.158 \times 10^7 \text{ s}$, 10^7)
- Find the number of times the human heart beats in the life of 60 years of a man, assuming that the heart beats once in 0.8 s. (Ans. $2.3668 \times 10^9 \text{ s}$)
- Two atomic clocks allowed to run for average life of an Indian (say, 70 years) differ by 0.2 s only. Calculate the accuracy of standard atomic clock in measuring a time interval of 1s. (Ans. 1 s in 10^{10} s)

- Age of the universe is about 10^{10} years whereas the mankind has existed for 10^6 years. For how many seconds would the man have existed if age of universe were 1 day ? (Ans. 8.64 s)

HINTS

- Here $t = 70 \text{ years} = 70 \times 365 + 17$

[17 days added due to 17 leap years in 70 years]

$$\text{or } t = 25567 \text{ days} = 25567 \times 86400 \text{ s}$$

$$\Delta t = 0.2 \text{ s}$$

Fractional error in time

$$= \frac{\Delta t}{t} = \frac{0.2}{25567 \times 86400} = 0.904 \times 10^{-10} \approx 10^{-10}$$

So the accuracy shown by atomic clock is 10^{-10} part in 1 s or 1 s in 10^{10} s .

- Magnification in time

$$= \frac{\text{Age of mankind}}{\text{Age of universe}} = \frac{10^6}{10^{10}} = 10^{-4}$$

Apparent age of mankind

$$= 10^{-4} \times 1 \text{ day} = 10^{-4} \times 86400 \text{ s} = 8.64 \text{ s.}$$

2.20 DIMENSIONS OF A PHYSICAL QUANTITY

40. What do you mean by seven dimensions of the world ?

Seven dimensions of the world. All the derived physical quantities can be expressed in terms of some combination of the seven fundamental or base quantities. We call these fundamental quantities as the seven dimensions of the world, which are denoted with square brackets [].

Dimension of length = [L]

Dimension of mass = [M]

Dimension of time = [T]

Dimension of electric current = [A]

Dimension of thermodynamic temperature = [K]

Dimension of luminous intensity = [cd]

Dimension of amount of substance = [mol]

41. What do you mean by dimensions of a physical quantity ? Explain with the help of an example.

Dimensions of a physical quantity. The dimensions of a physical quantity are the powers (or exponents) to which the fundamental quantities must be raised to represent that quantity completely.

For example,

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Length} \times \text{breadth} \times \text{height}}$$

∴ Dimensions of density

$$= \frac{[M]}{[L][L][L]} = [ML^{-3}] = [M^1L^{-3}T^0]$$

Hence the dimensions of density are '1' in mass, '-3' in length and '0' in time.

2.21 ▼ DIMENSIONAL FORMULAE AND DIMENSIONAL EQUATIONS

42. What is meant by dimensional formula and dimensional equation? Give examples.

Dimensional formula. The expression which shows how and which of the fundamental quantities represent the dimensions of a physical quantity is called the dimensional formula of the given physical quantity.

Examples : The dimensional formula of the volume is $[M^0L^3T^0]$ and that of momentum is $[MLT^{-1}]$.

Dimensional equation. The equation obtained by equating a physical quantity with its dimensional formula is called the dimensional equation of the given physical quantity.

Examples : The dimensional equation of force is

$$[\text{Force}] = [MLT^{-2}]$$

The dimensional equation for pressure is

$$[\text{Pressure}] = [ML^{-1}T^{-2}]$$

Dimensional formulae and SI units of some physical quantities are given in table 2.8.

Table 2.8 Dimensional Formulae and SI units of some physical quantities

S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
A. Mechanical Quantities				
1.	Area	Length × breadth	$L \times L = L^2 = [M^0L^2T^0]$	m^2
2.	Volume	Length × breadth × height	$L \times L \times L = [M^0L^3T^0]$	m^3
3.	Density	$\frac{\text{Mass}}{\text{Volume}}$	$\frac{M}{L^3} = [ML^{-3}T^0]$	$kg\ m^{-3}$
4.	Speed or velocity	$\frac{\text{Distance}}{\text{Time}}$	$\frac{L}{T} = [M^0LT^{-1}]$	ms^{-1}
5.	Acceleration	$\frac{\text{Change in velocity}}{\text{Time}}$	$\frac{LT^{-1}}{T} = LT^{-2} = [M^0LT^{-2}]$	ms^{-2}
6.	Momentum	Mass × velocity	$M \times LT^{-1} = [MLT^{-1}]$	$kg\ ms^{-1}$
7.	Force	Mass × acceleration	$M \times LT^{-2} = [MLT^{-2}]$	N
8.	Work	Force × distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	J
9.	Energy	Amount of work	$[ML^2T^{-2}]$	J
10.	Power	$\frac{\text{Work}}{\text{Time}}$	$\frac{ML^2T^{-2}}{T} = [ML^2T^{-3}]$	W
11.	Pressure	$\frac{\text{Force}}{\text{Area}}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Pa or Nm^{-2}
12.	Moment of force or torque	Force × $\perp$ distance	$MLT^{-2} \times L = [ML^2T^{-2}]$	Nm
13.	Gravitational constant 'G'	$\frac{\text{Force} \times (\text{distance})^2}{\text{Mass} \times \text{mass}}$	$\frac{MLT^{-2}L^2}{M \times M} = [M^{-1}L^3T^{-2}]$	Nm^2kg^{-2}
14.	Impulse of a force	Force × time	$MLT^{-2} \times T = [MLT^{-1}]$	Ns
15.	Stress	$\frac{\text{Force}}{\text{Area}}$	$\frac{MLT^{-2}}{L^2} = [ML^{-1}T^{-2}]$	Nm^{-2}
16.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[M^0L^0T^0]$ (dimensionless)	—
17.	Coefficient of elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$\frac{ML^{-1}T^{-2}}{1} = [ML^{-1}T^{-2}]$	Nm^{-2}

S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
18.	Surface tension	$\frac{\text{Force}}{\text{Length}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	Nm ⁻¹
19.	Surface energy	$\frac{\text{Work}}{\text{Area}}$	$\frac{ML^2T^{-2}}{L^2} = MT^{-2} = [ML^0T^{-2}]$	Jm ⁻²
20.	Coefficient of viscosity	$\frac{\text{Force} \times \text{distance}}{\text{Area} \times \text{velocity}}$	$\frac{MLT^{-2} \times L}{L^2 \times LT^{-1}} = [ML^{-1}T^{-1}]$	Nm ⁻² s or Pa s
21.	Angle	$\frac{\text{Arc}}{\text{Radius}}$	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)	rad
22.	Angular velocity	$\frac{\text{Angle}}{\text{Time}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	rad s ⁻¹
23.	Angular acceleration	$\frac{\text{Angular velocity}}{\text{Time}}$	$\frac{T^{-1}}{T} = T^{-2} = [M^0L^0T^{-2}]$	rad s ⁻²
24.	Moment of inertia	Mass $\times$ (distance) ²	$ML^2 = [ML^2T^0]$	kg m ²
25.	Radius of gyration	Distance	$L = [M^0LT^0]$	m
26.	Angular momentum	Mass $\times$ velocity $\times$ radius	$M \times LT^{-1} \times L = [ML^2T^{-1}]$	kg m ² s ⁻¹
27.	T-ratios (sin θ , cos θ , tan θ)	$\frac{\text{Length}}{\text{Length}}$	$\frac{L}{L} = 1 = [M^0L^0T^0]$ (dimensionless)	—
28.	Time period	Time	$T = [M^0L^0T]$	s
29.	Frequency	$\frac{1}{\text{Time period}}$	$\frac{1}{T} = T^{-1} = [M^0L^0T^{-1}]$	s ⁻¹ or Hz
30.	Planck's constant 'h'	$\frac{E}{\nu} = \frac{\text{Energy}}{\text{Frequency}}$	$\frac{ML^2T^{-2}}{T^{-1}} = [ML^2T^{-1}]$	Js
31.	Relative density	$\frac{\text{Density of substance}}{\text{Density of water at 4}^\circ\text{C}}$	$\frac{ML^{-3}}{ML^{-3}} = 1 = [M^0L^0T^0]$ (dimensionless)	—
32.	Velocity gradient	$\frac{\text{Velocity}}{\text{Distance}}$	$\frac{LT^{-1}}{L} = T^{-1} = [M^0L^0T^{-1}]$	s ⁻¹
33.	Pressure gradient	$\frac{\text{Pressure}}{\text{Distance}}$	$\frac{ML^{-1}T^{-2}}{L} = [ML^{-2}T^{-2}]$	Pa m ⁻¹
34.	Force constant	$\frac{\text{Force}}{\text{Displacement}}$	$\frac{MLT^{-2}}{L} = MT^{-2} = [ML^0T^{-2}]$	Nm ⁻¹
B. Thermal Quantities				
35.	Heat or enthalpy	Energy	$[ML^2T^{-2}]$	J
36.	Specific heat	$\frac{\text{Heat}}{\text{Mass} \times \text{Temperature}}$	$\frac{ML^2T^{-2}}{M \cdot K} = [M^0L^2T^{-2}K^{-1}]$	J kg ⁻¹ K ⁻¹
37.	Latent heat	$\frac{\text{Heat}}{\text{Mass}}$	$\frac{ML^2T^{-2}}{M} = [M^0L^2T^{-2}]$	J kg ⁻¹
38.	Thermal conductivity	$\frac{\text{Heat} \times \text{distance}}{\text{Area} \times \text{temp.} \times \text{time}}$	$\frac{ML^2T^{-2} \cdot L}{L^2 \cdot K \cdot T} = [MLT^{-3}K^{-1}]$	Js ⁻¹ m ⁻¹ K ⁻¹
39.	Entropy	$\frac{\text{Heat}}{\text{Temperature}}$	$\frac{ML^2T^{-2}}{K} = [ML^2T^{-2}K^{-1}]$	JK ⁻¹
40.	Universal gas constant	$\frac{PV}{nT}$	$\frac{ML^{-1}T^{-2}L^3}{\frac{\text{mol} \cdot K}{[ML^2T^{-2}K^{-1} \text{ mol}^{-1}]}}$	J mol ⁻¹ K ⁻¹

S. No.	Physical Quantity	Relation with other quantities	Dimensional formula	SI unit
41.	Boltzmann's constant	$\frac{\text{Energy}}{\text{Temperature}}$	$\frac{ML^2 T^{-2}}{K} = [ML^2 T^{-2} K^{-1}]$	JK^{-1}
42.	Stefan's constant	$\frac{\text{Energy}}{\text{Area} \times \text{time} \times (\text{temp.})^4}$	$\frac{ML^2 T^{-2}}{L^2 \cdot T \cdot K^4} = [ML^0 T^{-3} K^{-4}]$	$Js^{-1} m^{-2} K^{-4}$
43.	Solar constant	$\frac{\text{Energy}}{\text{Area} \times \text{time}}$	$\frac{ML^2 T^{-2}}{L^2 \cdot T} = [ML^0 T^{-3}]$	$Js^{-1} m^{-2}$
44.	Mechanical equivalent of heat	$J = \frac{W}{H}$	$\frac{ML^2 T^{-2}}{ML^2 T^{-2}} = [M^0 L^0 T^0]$ (dimensionless)	—
C. Electrical Quantities				
45.	Electric charge	Time $\times$ Current	$T \cdot A = [M^0 L^0 TA]$	C (coulomb)
46.	Electrical potential	$\frac{\text{Work}}{\text{Charge}}$	$\frac{ML^2 T^{-2}}{TA} = [ML^2 T^{-3} A^{-1}]$	V (volt)
47.	Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{ML^2 T^{-3} A^{-1}}{A} = [ML^2 T^{-3} A^{-2}]$	Ω (ohm)
48.	Capacitance	$\frac{\text{Charge}}{\text{Potential difference}}$	$\frac{TA}{ML^2 T^{-3} A^{-1}} = [M^{-1} L^{-2} T^4 A^2]$	F (farad)
49.	Inductance	$\frac{\text{EMF}}{\text{Current / time}}$	$\frac{ML^2 T^{-3} A^{-1}}{AT^{-1}} = [ML^2 T^{-2} A^{-2}]$	H (henry)
50.	Permittivity of free space	$\epsilon_0 = \frac{q_1 q_2}{Fr^2}$	$\frac{AT \cdot AT}{MLT^{-2} L^2} = [M^{-1} L^{-3} T^4 A^2]$	$A^2 C^2 N^{-1} m^{-2}$
51.	Relative permittivity or dielectric constant	$\epsilon_r \text{ or } \kappa = \frac{\epsilon_0}{\epsilon}$	a pure ratio = $[M^0 L^0 T^0]$ (dimensionless)	—
52.	Intensity of electric field	$E = \frac{F}{q} = \frac{\text{Force}}{\text{Charge}}$	$\frac{MLT^{-2}}{AT} = [MLT^{-3} A^{-1}]$	NC^{-1} or Vm^{-1}
53.	Conductance	$C = \frac{1}{R}$	$\frac{1}{ML^2 T^{-3} A^{-2}} = [M^{-1} L^{-2} T^3 A^2]$	Ω^{-1} or mho
54.	Specific resistance or resistivity	$\rho = \frac{RA}{l}$	$\frac{ML^2 T^{-3} A^{-2} \cdot L^2}{L} = [ML^3 T^{-3} A^{-2}]$	Ωm
55.	Specific conductance or conductivity	$\sigma = \frac{1}{\rho}$	$[M^{-1} L^{-3} T^3 A^2]$	$\Omega^{-1} m^{-1}$
56.	Electric dipole moment	$q \times 2l$	$AT \cdot L = [M^0 LTA]$	Cm
D. Magnetic Quantities				
57.	Magnetic field	$B = \frac{F}{qv \sin \theta}$	$\frac{MLT^{-2}}{AT \cdot LT^{-1} \cdot 1} = [ML^0 T^{-2} A^{-1}]$	T (tesla)
58.	Magnetic flux	$\phi = BA$	$MT^{-2} A^{-1} \cdot L^2 = [ML^2 T^{-2} A^{-1}]$	Wb (weber)
59.	Permeability of free space	$\mu_0 = \frac{4\pi r \cdot F}{I_1 I_2 l}$	$\frac{L \cdot MLT^{-2}}{A^2 \cdot L} = [MLT^{-2} A^{-2}]$	
60.	Magnetic moment	Current $\times$ area	$A \cdot L^2 = [M^0 L^2 T^0 A]$	Am^2
61.	Pole strength	$\frac{\text{Magnetic moment}}{\text{Magnetic length}}$	$\frac{AL^2}{L} = [M^0 LT^0 A]$	Am

Examples based on Derivation of Dimensional Formulae

CONCEPT USED

The dimensional formula of a physical quantity can be obtained by defining its relation with other quantities, whose dimensions in M, L and T are known.

EXAMPLE 27. Name the physical quantities whose dimensional formulae are as follows :

- (i) $ML^2 T^{-2}$ (ii) $ML^2 T^{-3}$ (iii) MT^{-2}
(iv) $ML^{-1} T^{-1}$ (v) $ML^{-1} T^{-2}$.

Solution. (i) $ML^2 T^{-2} = MLT^{-2} \cdot L$
 $= \text{Force} \times \text{distance} = \text{Work}.$

(ii) $ML^2 T^{-3} = \frac{ML^2 T^{-2}}{T} = \frac{\text{Work}}{\text{Time}} = \text{Power}.$

(iii) $MT^{-2} = \frac{MLT^{-2}}{L} = \frac{\text{Force}}{\text{Length}}$
 $= \text{Surface tension or force constant}.$

(iv) $ML^{-1} T^{-1} = \frac{MLT^{-2}}{L^2 T^{-1}} = \frac{\text{Force}}{\text{Area} \times \text{velocity gradient}}$
 $= \text{Coefficient of viscosity}.$

(v) $ML^{-1} T^{-2} = \frac{MLT^{-2}}{L^2} = \frac{\text{Force}}{\text{Area}}$
 $= \text{Pressure or stress}.$

EXAMPLE 28. Deduce the dimensional formulae for the following physical quantities :

- (i) Gravitational constant [Delhi 13]
(ii) Power (iii) Young's modulus
(iv) Coefficient of viscosity (v) Surface tension
(vi) Planck's constant.

Solution. (i) According to Newton's law of gravitation,

$$F = G \frac{m_1 m_2}{r^2}$$

$$\therefore [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{MLT^{-2} \cdot L^2}{MM} = [M^{-1} L^3 T^{-2}].$$

(ii) $\text{Power} = \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Distance}}{\text{Time}}$

$$\therefore [\text{Power}] = \frac{MLT^{-2} \cdot L}{T} = [ML^2 T^{-3}].$$

(iii) Young's modulus,

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F/A}{\Delta l/l} = \frac{F}{A} \cdot \frac{l}{\Delta l}$$

$$\therefore [Y] = \frac{MLT^{-2} \cdot L}{L^2 \cdot L} = [ML^{-1} T^{-2}].$$

(iv) Coefficient of viscosity,

$$\eta = \frac{\text{Force}}{\text{Area} \times \text{velocity gradient}} = \frac{\text{Force}}{\text{Area}} \times \frac{\text{Distance}}{\text{Velocity}}$$

$$[\eta] = \frac{MLT^{-2} \cdot L}{L^2 \cdot LT^{-1}} = [ML^{-1} T^{-1}].$$

(v) Surface tension,

$$\sigma = \frac{\text{Force}}{\text{Length}}$$

$$[\sigma] = \frac{MLT^{-2}}{L} = [MT^{-2}].$$

(vi) Planck's constant,

$$h = \frac{\text{Energy}}{\text{Frequency}}$$

$$[h] = \frac{ML^2 T^{-2}}{T^{-1}} = [ML^2 T^{-1}].$$

EXAMPLE 29. Deduce the dimensional formulae of the following physical quantities :

- (i) Heat (ii) Specific heat (iii) Latent heat
(iv) Gas constant
(v) Boltzmann's constant
(vi) Coefficient of thermal conductivity
(vii) Mechanical equivalent of heat.

Solution. (i) Heat = Energy

$$\therefore [\text{Heat}] = [ML^2 T^{-2}].$$

(ii) Specific heat = $\frac{\text{Heat}}{\text{Mass} \times \text{temperature}}$

$$\therefore [\text{Specific heat}] = \frac{ML^2 T^{-2}}{M \cdot K} = [L^2 T^{-2} K^{-1}].$$

(iii) Latent heat = $\frac{\text{Heat}}{\text{Mass}}$

$$\therefore [\text{Latent heat}] = \frac{ML^2 T^{-2}}{M} = [L^2 T^{-2}].$$

(iv) $PV = nRT$

or $R = \frac{PV}{nT} = \frac{\text{Force} \times \text{volume}}{\text{mol} \times \text{area} \times \text{temp.}}$

$$\therefore [R] = \frac{MLT^{-2} \cdot L^3}{\text{mol} \cdot L^2 \cdot K} = [ML^2 T^{-2} K^{-1} \text{mol}^{-1}].$$

(v) Boltzmann's constant,

$$k = \frac{\text{Heat}}{\text{Temperature}}$$

$$[k] = \frac{ML^2 T^{-2}}{K} = [ML^2 T^{-2} K^{-1}].$$

(vi) Coefficient of thermal conductivity,

$$\kappa = \frac{\text{Heat} \times \text{distance}}{\text{Area} \times \text{time} \times \text{temperature difference}}$$

$$[\kappa] = \frac{\text{ML}^2\text{T}^{-2} \cdot \text{L}}{\text{L}^2 \cdot \text{T} \cdot \text{K}} = [\text{MLT}^{-3}\text{K}^{-1}]$$

(vii) Joule's mechanical equivalent of heat,

$$J = \frac{\text{Work}}{\text{Heat}}$$

$$[J] = \frac{\text{ML}^2\text{T}^{-2}}{\text{ML}^2\text{T}^{-2}} = 1 = [\text{M}^0\text{L}^0\text{T}^0]$$

EXAMPLE 30. Find the dimensional formulae of (i) charge (ii) potential (iii) resistance (iv) capacitance.**Solution.** (i) Charge, $q = \text{Current} \times \text{time}$

$$\therefore [q] = [\text{AT}]$$

(ii) Potential,

$$V = \frac{\text{Work}}{\text{Charge}}$$

$$[V] = \frac{\text{ML}^2\text{T}^{-2}}{\text{AT}} = [\text{ML}^2\text{A}^{-1}\text{T}^{-3}]$$

(iii) Resistance,

$$R = \frac{\text{Potential difference}}{\text{Current}} = \frac{\text{ML}^2\text{A}^{-1}\text{T}^{-3}}{\text{A}} = [\text{ML}^2\text{A}^{-2}\text{T}^{-3}]$$

(iv) Capacitance,

$$C = \frac{\text{Charge}}{\text{Potential}}$$

$$[C] = \frac{\text{AT}}{\text{ML}^2\text{A}^{-1}\text{T}^{-3}} = [\text{M}^{-1}\text{L}^{-2}\text{A}^2\text{T}^4]$$

EXAMPLE 31. Taking velocity, time and force as the fundamental quantities, find the dimensions of mass.**Solution.** Force = Mass $\times$ Acceleration

$$= \text{Mass} \times \frac{\text{Velocity}}{\text{Time}}$$

$$\text{Mass} = \frac{\text{Force} \times \text{Time}}{\text{Velocity}}$$

$$[\text{Mass}] = [\text{FTV}^{-1}]$$

EXAMPLE 32. If density ρ , acceleration due to gravity g and frequency ν are the basic quantities, find the dimensions of force.**Solution.** We have $\rho = \text{ML}^{-3}$, $g = \text{LT}^{-2}$, $\nu = \text{T}^{-1}$ Solving for M, L and T in terms of ρ , g and ν , we get

$$M = \rho g^3 \nu^{-6}, \quad L = g \nu^{-2}, \quad T = \nu^{-1}$$

$$[\text{Force}] = \text{MLT}^{-2} = \rho g^3 \nu^{-6} \cdot g \nu^{-2} \cdot \nu^2 = [\rho g^4 \nu^{-6}]$$

EXAMPLE 33. If the velocity of light c , acceleration due to gravity g and atmospheric pressure p are the fundamental quantities, find the dimensions of length.**Solution.** We have,

$$c = \text{LT}^{-1}, \quad g = \text{LT}^{-2}, \quad p = \text{ML}^{-1}\text{T}^{-2}$$

$$[\text{Length}] = L = \frac{\text{L}^2\text{T}^{-2}}{\text{LT}^{-2}} = \frac{(\text{LT}^{-1})^2}{\text{LT}^{-2}} = \left[\frac{c^2}{g} \right]$$

EXAMPLE 34. The number of particles crossing a unit area perpendicular to X-axis in unit time is given by

$$n = -D \frac{n_2 - n_1}{x_2 - x_1}$$

where n_1 and n_2 are number of particles per unit volume for the values of x meant to be x_1 and x_2 . Find the dimensions of the diffusion constant D .

$$\text{Solution. As } n = -D \frac{n_2 - n_1}{x_2 - x_1}$$

$$\therefore D = \frac{n(x_2 - x_1)}{(n_2 - n_1)} \quad (\text{numerically})$$

Now n = number of particles per unit area per second,

$$\therefore [n] = \text{L}^{-2}\text{T}^{-1}$$

 $n_2 - n_1$ = number of particles per unit volume

$$\therefore [n_2 - n_1] = \text{L}^{-3}$$

 $x_2 - x_1$ = position

$$\therefore [x_2 - x_1] = \text{L}$$

$$\text{Hence } [D] = \frac{\text{L}^{-2}\text{T}^{-1} \cdot \text{L}}{\text{L}^{-3}} = [\text{L}^2\text{T}^{-1}]$$

PROBLEMS FOR PRACTICE

1. Deduce dimensional formulae for (i) angle (ii) angular velocity (iii) angular acceleration (iv) torque (v) angular momentum and (vi) moment of inertia.

$$[\text{Ans. (i) Dimensionless (ii) T}^{-1} \text{ (iii) T}^{-2} \text{ (iv) ML}^2\text{T}^{-2} \text{ (v) ML}^2\text{T}^{-1} \text{ (vi) ML}^2]$$

2. Obtain dimensions of (i) impulse (ii) power (iii) surface energy (iv) coefficient of viscosity (v) bulk modulus (vi) force constant.

$$[\text{Ans. (i) MLT}^{-1} \text{ (ii) ML}^2\text{T}^{-3} \text{ (iii) ML}^0\text{T}^{-2} \text{ (iv) ML}^{-1}\text{T}^{-1} \text{ (v) ML}^{-1}\text{T}^{-2} \text{ (vi) ML}^0\text{T}^{-2}]$$

3. By the use of dimensions, show that energy per unit volume is equal to the pressure.
4. Show that angular momentum has the same physical units as the Planck's constant h which is given by the relation $E = h\nu$.
5. If force (F), length (L) and time (T) are chosen as the fundamental quantities, then what would be the dimensional formula for density? (Ans. $\text{FL}^{-4}\text{T}^{-1}$)

6. Calculate the dimensions of force and impulse taking velocity, density and frequency as basic quantities.
(Ans. $\rho v^4 v^{-2}$, $\rho v^4 v^{-3}$)

7. Find the dimensions of linear momentum and surface tension in terms of velocity v , density ρ and frequency ν as fundamental quantities.

(Ans. $\rho v^4 v^{-3}$, $\rho v^3 v^{-1}$)

8. In the expression $P = El^2 m^{-5} G^{-2}$; E , m , l and G denote energy, mass angular momentum and gravitational constant, respectively. Show that P is a dimensionless quantity. [Exemplar Problem]

HINTS

3. Energy per unit volume

$$\begin{aligned} &= \frac{\text{Energy}}{\text{Volume}} = \frac{ML^2 \cdot T^{-2}}{L^3} = \frac{MLT^{-2}}{L^2} \\ &= \frac{\text{Force}}{\text{Area}} = \text{Pressure.} \end{aligned}$$

4. Angular momentum,

$$L = mvr$$

$$[L] = MLT^{-1} \cdot L = ML^2T^{-1}$$

As $E = hv$

$$h = \frac{E}{\nu}$$

$$[h] = \frac{ML^2 T^{-2}}{T^{-1}} = ML^2 T^{-1} = [L]$$

6. $v = LT^{-1}$, $\rho = ML^{-3}$, $\nu = T^{-1}$

Solving for M , L and T in terms of ν , ρ and v , we get

$$T = v^{-1}, L = v\nu^{-1}, M = \rho v^3 \nu^{-3}$$

$$[\text{Force}] = MLT^{-2} = \rho v^3 \nu^{-3} \cdot v\nu^{-1} \cdot v^2 = \rho v^4 \nu^{-2}$$

$$[\text{Impulse}] = \text{Force} \times \text{time} = \rho v^4 \nu^{-2} \cdot \nu^{-1} = \rho v^4 \nu^{-3}$$

7. Using the dimensions of M , L and T as obtained in Problem 6, we get

$$\begin{aligned} [p] &= [mv] = MLT^{-1} \\ &= \rho v^3 \nu^{-3} \cdot v\nu^{-1} \cdot v = \rho v^4 \nu^{-3} \end{aligned}$$

$$\text{Surface tension} = \frac{\text{Force}}{\text{Length}}$$

$$= \frac{MLT^{-2}}{L} = MT^{-2} = \rho v^3 \nu^{-3} \nu^2 = \rho v^3 \nu^{-1}$$

8. $[E] = ML^2 T^{-2}$, $[M] = M$, $[l] = ML^2 T^{-1}$,

$$[G] = M^{-1} L^3 T^{-2}$$

$$\begin{aligned} \therefore \left[\frac{El^2}{M^5 G^2} \right] &= \frac{ML^2 T^{-2} [ML^2 T^{-1}]^2}{M^5 [M^{-1} L^3 T^{-2}]^2} \\ &= \frac{M^3 L^6 T^{-4}}{M^3 L^6 T^{-4}} = [M^0 L^0 T^0] \end{aligned}$$

2.22 DIFFERENT TYPES OF VARIABLES AND CONSTANTS

43. How can we classify variables and constants on the basis of dimensions? Give examples of each type.

Different types of variables and constants. On the basis of dimensions, we can classify quantities into four categories:

1. **Dimensional variables.** The physical quantities which possess dimensions and have variable values are called dimensional variables.

Examples. Area, volume, velocity, force, etc.

2. **Dimensionless variables.** The physical quantities which have no dimensions but have variable values are called dimensionless variables.

Examples. Angle, specific gravity, strain, etc.

3. **Dimensional constants.** The physical quantities which possess dimensions and have constant values are called dimensional constants.

Examples. Gravitational constant, Planck's constant, etc.

4. **Dimensionless constants.** The constant quantities having no dimensions are called dimensionless constants.

Examples. π , e , etc.

2.23 APPLICATIONS OF DIMENSIONAL ANALYSIS

44. Mention some applications of dimensional analysis.

Applications of dimensional analysis. The method of studying a physical phenomenon on the basis of dimensions is called dimensional analysis. Following are the three main uses of dimensional analysis:

1. To convert a physical quantity from one system of units to another.
2. To check the correctness of a given physical relation.
3. To derive a relationship between different physical quantities.

2.24 CONVERSION OF ONE SYSTEM OF UNITS TO ANOTHER

45. How can a physical quantity be converted from one system of units to another? Explain it with the help of a suitable example.

To convert a physical quantity from one system of units to another. It is based on the fact that the magnitude of a physical quantity remains the same, whatever may be the system of units. If u_1 and u_2 are the units of measurement of a physical quantity Q and n_1 and n_2 are the corresponding numerical values, then

$$Q = n_1 u_1 = n_2 u_2$$

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Conversion of one system of units
to another system of units

$$q = n_1 u_1 = n_2 u_2 \quad \text{--- (1)}$$

Let $M_1, L_1, T_1 \rightarrow$ one system [size of fundamental units].

$M_2, L_2, T_2 \rightarrow$ another system

$M^a L^b T^c$

$$u_1 = M_1^a L_1^b T_1^c$$

$$u_2 = M_2^a L_2^b T_2^c$$

$$\therefore \text{from (1)} \quad n_1 M_1^a L_1^b T_1^c = n_2 M_2^a L_2^b T_2^c$$

$$\therefore n_2 = n_1 \frac{M_1^a L_1^b T_1^c}{M_2^a L_2^b T_2^c}$$

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

Q Convert one joule into ergs?

Sol $\vec{E} = M^1 L^2 T^{-2}$

$a=1, b=2, c=-2$

energy
Joule erg
| |
S.I. $\rightarrow$ C.G.S.

S.I	C.G.S.
$M_1 = 1 \text{ Kg}$	$M_2 = 1 \text{ g}$
$L_1 = 1 \text{ m}$	$L_2 = 1 \text{ cm}$
$T_1 = 1 \text{ s}$	$T_2 = 1 \text{ s}$
$n_1 = 1$	$n_2 = ?$

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we know $n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$

$$= 1 \left[\frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[\frac{1 \text{ m}}{1 \text{ cm}} \right]^2 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2}$$

$$= 1 \left[\frac{1000 \text{ g}}{1 \text{ g}} \right]^1 \left[\frac{100 \text{ cm}}{1 \text{ cm}} \right]^2 \left[\frac{1}{1} \right]^{-2}$$

$$n_2 = 1 \times (1000)^1 \times (100)^2 \times 1 = 10^3 \times 10^4$$

$$n_2 = 10^{5+2} = 10^7$$

$$n_1 u_1 = n_2 u_2$$

$$\boxed{1 \text{ J} = 10^7 \text{ erg}}$$

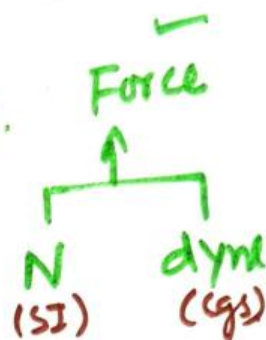
Q Convert one newton into dyne.

$$F = M^1 L^1 T^{-2}$$

$$a=1, b=1, c=-2$$

SI	cgs.
$M_1 = 1\text{kg}$	$M_2 = 1\text{g}$
$L_1 = 1\text{m}$	$L_2 = 1\text{cm}$
$T_1 = 1\text{s}$	$T_2 = 1\text{s}$
$\eta_1 = 1\text{N}$	$\eta_2 = ? \dots \text{dyne}$

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$$\eta_2 = \eta_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$= 1 \left[\frac{1000\text{g}}{1\text{g}} \right]^1 \left[\frac{100\text{cm}}{1\text{cm}} \right]^1 \left[\frac{1\text{s}}{1\text{s}} \right]^{-2}$$

$$= 1 \times (1000)^1 \times (100)^1 \times 1$$

$$\eta_2 = 10^5$$

$$\therefore \boxed{1\text{N} = 10^5 \text{dyne}}$$

Q The value of G in CGS system 6.67×10^{-8} dyne $\text{cm}^2 \text{g}^{-2}$. Calculate its value in SI system.

sol $[G] = M^{-1} L^3 T^{-2}$.

$a = -1, b = 3, c = -2$.



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CGS	SI
$M_1 = 1 \text{g}$	$M_2 = 1 \text{kg}$
$L_1 = 1 \text{cm}$	$L_2 = 1 \text{m}$
$T_1 = 1 \text{s}$	$T_2 = 1 \text{s}$
$\eta_1 = 6.67 \times 10^{-8}$	$\eta_2 = ?$

$$\eta_2 = \eta_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$= 6.67 \times 10^{-8} \left[\frac{1 \text{g}}{1 \text{kg}} \right]^{-1} \left[\frac{1 \text{cm}}{1 \text{m}} \right]^3 \left[\frac{1 \text{s}}{1 \text{s}} \right]^{-2}$$

$$\eta_2 = 6.67 \times 10^{-8} \left[\frac{1 \text{g}}{1000 \text{g}} \right]^{-1} \left[\frac{1 \text{cm}}{100 \text{cm}} \right]^3 \left[\frac{1 \text{s}}{1 \text{s}} \right]^{-2}$$

$$= 6.67 \times 10^{-8} \left[\frac{1}{1000} \right]^{-1} \left[\frac{1}{100} \right]^3 \left[\frac{1}{1} \right]^{-2}$$

$$= 6.67 \times 10^{-8} (1000)^1 [100]^{-3} [1]^2$$

$$= 6.67 \times 10^{-8} \times 10^3 \times 10^{-6} \times 1$$

$$\eta_2 = 6.67 \times 10^{-11}$$

$$6.67 \times 10^{-8} \text{ dyne cm}^2 \text{g}^{-2} = 6.67 \times 10^{-11} \text{ Nm}^2 \text{kg}^{-2}$$

Signature

Q2 Find the value of 1W on a system that has 100g, 100cm and 1min as the base units.

W → Power

sol $P = [M^1 L^2 T^{-3}]$

$a=1, b=2, c=-3.$

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S.I	New System
$M_1 = 1\text{Kg}$	$M_2 = 100\text{g}$
$L_1 = 1\text{m}$	$L_2 = 100\text{cm}$
$T_1 = 1\text{s}$	$T_2 = 1\text{min}$
$\eta_1 = 1\text{W}$	$\eta_2 = ?$

$$\eta_2 = \eta_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$\eta_2 = 1 \left[\frac{1\text{Kg}}{100\text{g}} \right]^1 \left[\frac{1\text{m}}{100\text{cm}} \right]^2 \left[\frac{1\text{s}}{1\text{min}} \right]^{-3}$$

$$= 1 \left[\frac{1000\text{g}}{100\text{g}} \right]^1 \left[\frac{100\text{cm}}{100\text{cm}} \right]^2 \left[\frac{1\text{s}}{60\text{s}} \right]^{-3}$$

$$= 1 \times 10^1 \times 1^2 \times 60^3$$

$$= 10 \times 216000 = 2160000$$

$$\eta_2 = 2.16 \times 10^6$$

$$\therefore 1\text{W} = 2.16 \times 10^6 \text{ new units of power.}$$

Q Find the value of 100j on a system which has 20cm, 250g & half minute as fundamental units of length, mass & time.

Sol $W = E = M^1 L^2 T^{-2}$

$a=1, b=2, c=-2$

S.I	New System
$M_1 = 1\text{kg}$	$M_2 = 250\text{g}$
$L_1 = 1\text{m}$	$L_2 = 20\text{cm}$
$T_1 = 1\text{s}$	$T_2 = 0.5\text{min}$
$\eta_1 = 100\text{j}$	$\eta_2 = ?$

★

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$$\eta_2 = \eta_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

$$= 100 \left[\frac{1\text{kg}}{250\text{g}} \right]^1 \left[\frac{1\text{m}}{20\text{cm}} \right]^2 \left[\frac{1\text{s}}{0.5\text{min}} \right]^{-2}$$

$$= 100 \left[\frac{1000\cancel{\text{g}}}{250\cancel{\text{g}}} \right]^1 \left[\frac{100\cancel{\text{cm}}}{20\cancel{\text{cm}}} \right]^2 \left[\frac{1\cancel{\text{s}}}{30\cancel{\text{s}}} \right]^{-2}$$

$$= 100 \times 4 \times 25 \times 30 \times 30$$

$$= 9 \times 10^6$$

$$\therefore \boxed{100\text{j} = 9 \times 10^6 \text{ new unit}}$$

EXAMPLE 37. In CGS system, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$. Find its value in SI units. Given $1 \text{ J} = 10^7 \text{ erg}$.

Solution. In CGS system, $\sigma = 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$. The SI unit of work is joule. We have, $1 \text{ erg} = 10^{-7} \text{ J}$ and $1 \text{ cm} = 10^{-2} \text{ m}$

$\therefore$ The value of Stefan's constant in SI units is

$$\begin{aligned}\sigma &= 5.67 \times 10^{-5} [10^{-7} \text{ J}] \text{ s}^{-1} [10^{-2} \text{ m}]^{-2} \text{ K}^{-4} \\ &= 5.67 \times 10^{-5} \times 10^{-7} \times 10^4 \text{ Js}^{-1} \text{ m}^{-2} \text{ K}^{-4} \\ &= 5.67 \times 10^{-8} \text{ Js}^{-1} \text{ m}^{-2} \text{ K}^{-4}.\end{aligned}$$

EXAMPLE 38. If the unit of force is 1 kN, unit of length 1 km and the unit of time is 100 s, what will be the unit of mass?

Solution. $M = \frac{MLT^{-2} \cdot T^2}{L} = \frac{FT^2}{L}$

$$\begin{aligned}&= \frac{1000 \text{ N} \times 10^4 \text{ s}^2}{1000 \text{ m}} = 10^4 \frac{\text{Ns}^2}{\text{m}} \\ &= 10^4 \frac{\text{kg m s}^{-2} \cdot \text{s}^2}{\text{m}} = 10^4 \text{ kg}.\end{aligned}$$

PROBLEMS FOR PRACTICE

- Convert one dyne into newton. [Himachal 09]
(Ans. 10^{-5} newton)
- If the value of universal gravitational constant in SI is $6.6 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$, then find its value in CGS system. [Himachal 09]
(Ans. $6.6 \times 10^{-8} \text{ dyne cm}^2 \text{ g}^{-2}$)
- The density of mercury is 13.6 g cm^{-3} in CGS system. Find its value in SI units.
(Ans. $13.6 \times 10^3 \text{ kg m}^{-3}$)
- The surface tension of water is 72 dyne cm^{-1} . Express it in SI units. (Ans. 0.072 Nm^{-1})
- An electric bulb has a power of 500 W. Express it in CGS units. (Ans. $5 \times 10^9 \text{ erg s}^{-1}$)
- If the value of atmospheric pressure is $10^6 \text{ dyne cm}^{-2}$, find its value in SI units. (Ans. 10^5 Nm^{-2})
- In SI units, the value of Stefan's constant is $\sigma = 5.67 \times 10^{-8} \text{ Js}^{-1} \text{ m}^{-2} \text{ K}^{-4}$. Find its value in CGS system. (Ans. $5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}$)
- Find the value of 100 J on a system which has 20 cm, 250 g and half minute as fundamental units of length, mass and time. (Ans. 9×10^6 new units)
- If the units of force, energy and velocity are 20 N, 200 J and 5 ms^{-1} , find the units of length, mass and time. (Ans. 10 m, 8 kg, 2 s)
- When 1 m, 1 kg and 1 min are taken as the fundamental units, the magnitude of the force is 36 units. What will be the value of this force in CGS system?
(Ans. 10^3 dyne)

HINTS

3. [Density] = $M^1 L^{-3}$

$$\begin{aligned}n_2 &= n_1 \left[\frac{M_1}{M_2} \right]^1 \left[\frac{L_1}{L_2} \right]^{-3} \\ &= 13.6 \left[\frac{1 \text{ g}}{1 \text{ kg}} \right]^1 \left[\frac{1 \text{ cm}}{1 \text{ m}} \right]^{-3} \\ &= 13.6 \left[\frac{1 \text{ g}}{1000 \text{ g}} \right] \left[\frac{1 \text{ cm}}{100 \text{ cm}} \right]^{-3} = 13.6 \times 10^3\end{aligned}$$

$$\therefore 13.6 \text{ g cm}^{-3} = 13.6 \times 10^3 \text{ kg m}^{-3}.$$

4. [Surface Tension] = $M^1 T^{-2}$

$$\begin{aligned}n_2 &= 72 \left[\frac{1 \text{ g}}{1 \text{ kg}} \right]^1 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2} \\ &= 72 \left[\frac{1 \text{ g}}{1000 \text{ g}} \right]^1 \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2} = 0.072\end{aligned}$$

$$\therefore 72 \text{ dyne cm}^{-1} = 0.072 \text{ Nm}^{-1}.$$

6. [Pressure] = $M^1 L^{-1} T^{-2}$

$$n_2 = 10^6 \left[\frac{1 \text{ g}}{1 \text{ kg}} \right]^1 \left[\frac{1 \text{ cm}}{1 \text{ m}} \right]^{-1} \left[\frac{1 \text{ s}}{1 \text{ s}} \right]^{-2} = 10^5$$

$$\therefore 10^6 \text{ dyne cm}^{-2} = 10^5 \text{ Nm}^{-2}.$$

7. $\sigma = 5.67 \times 10^{-8} \text{ Js}^{-1} \text{ m}^{-2} \text{ K}^{-4}$

$$\begin{aligned}&= 5.67 \times 10^{-8} \times 10^7 \text{ erg s}^{-1} (100 \text{ cm})^{-2} \text{ K}^{-4} \\ &= 5.67 \times 10^{-5} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ K}^{-4}.\end{aligned}$$

8. [Work] = $ML^2 T^{-2}$

$$\begin{aligned}n_2 &= 100 \left[\frac{1 \text{ kg}}{250 \text{ g}} \right]^1 \left[\frac{1 \text{ m}}{20 \text{ cm}} \right]^2 \left[\frac{1 \text{ s}}{0.5 \text{ min}} \right]^{-2} \\ &= 100 \left[\frac{1000 \text{ g}}{250 \text{ g}} \right]^1 \left[\frac{100 \text{ cm}}{20 \text{ cm}} \right]^2 \left[\frac{1 \text{ s}}{30 \text{ s}} \right]^{-2} \\ &= 100 \times 4 \times 25 \times 30 \times 30 = 9 \times 10^6 \text{ new units}.\end{aligned}$$

9. (i) $MLT^{-2} = 20 \text{ N}$ (ii) $ML^2 T^{-2} = 200 \text{ J}$

(iii) $LT^{-1} = 5 \text{ ms}^{-1}$

Dividing (ii) by (i), $L = \frac{200}{20} = 10 \text{ m}$

Putting the value of L in (iii),

$$10 T^{-1} = 5 \text{ or } T = 2 \text{ s}$$

From (i),

$$M \times 10 \times (2)^{-2} = 20 \text{ or } M = 8 \text{ kg}.$$

10. $n_2 = 36 \left[\frac{1 \text{ kg}}{1 \text{ g}} \right]^1 \left[\frac{1 \text{ m}}{1 \text{ cm}} \right]^1 \left[\frac{1 \text{ min}}{1 \text{ s}} \right]^{-2}$

$$= 36 \left[\frac{1000 \text{ g}}{1 \text{ g}} \right] \left[\frac{100 \text{ cm}}{1 \text{ cm}} \right]^1 \left[\frac{60 \text{ s}}{1 \text{ s}} \right]^{-2} = 10^3 \text{ dyne}.$$

Principle of homogeneity of Dimensions:

A physical eqn will be dimensionally correct if the dimensions of all the terms occurring on bds of the equation are same. ★✓

$$A + B = C - D$$

=n is Dim Correct

$$\text{if } \text{Dim } A = \text{Dim } B = \text{Dim } C = \text{Dim } D$$

$$\# \quad AB + C = D$$

$$\text{correct if } \text{Dim } AB = \text{Dim } C = \text{Dim } D$$

Checking the dimensional consistency of equations
or

Checking the correctness of eqns.

$$\text{eg. } S = ut + \frac{1}{2} at^2$$

$$L' = L' T^{-1} \times T^1 + L' T^{-2} (T^1)^2$$

$$L' = L' T^{-1+1} + L' T^{-2} T^2$$

$$L' = L' T^0 + L' T^0$$

$$\text{or } M^0 L' T^0 = M^0 L' T^0 + M^0 L' T^0$$

Dim of each term on bds is same.

$\therefore$ eqn is dimensionally correct.



eg

$$v^2 - u^2 = 2as$$

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$$(L^1 T^{-1})^2 - (L^1 T^{-1})^2 = L^1 T^{-2} \times L^1$$

$$L^2 T^{-2} - L^2 T^{-2} = L^2 T^{-2}$$

$\therefore$ Dim of each term on lds is same

$\therefore =n$ is dimensionally correct

eg check $FS^2 = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$\underline{\underline{Sol}} \quad M^1 L^1 T^{-2} \times L^2 = M^1 (L^1 T^{-1})^2 - M^1 (L^1 T^{-1})^2$$

$$M^1 L^3 T^{-2} = M^1 L^2 T^{-2} - M^1 L^2 T^{-2}$$

Since the Dim of each term on lds is not same

$\therefore =n$ is not dimensionally correct

$\Rightarrow$ Correct $=n$ is $FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

check $\frac{1}{2}mv^2 = \text{work}$

$$M^1 (L^1 T^{-1})^2 = M^1 \times L^1 T^{-2} \times L^1$$

$$M^1 L^2 T^{-2} = M^1 L^2 T^{-2}$$

$\therefore$ Dims of each term on lds is same

$\therefore =n$ is Dimnsly Correct

Q check $\lambda = \frac{h}{mv}$, where h plank's constant
 $\lambda = \text{wavelength (Distance)}$.

sq

$$L^1 = \frac{M^1 L^2 T^{-1}}{M^1 \times L^1 T^{-1}} = M^1 L^2 T^{-1} \times M^{-1} L^{-1} T^1$$

$$= M^{1-1} L^{2-1} T^{-1+1}$$

$$M^0 L^1 T^0 = M^0 L^1 T^0$$

$\Rightarrow$ is dimensionally correct

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Q check $T = 2\pi \sqrt{\frac{L}{g}}$.

$$M^0 L^0 T^1 = \sqrt{\frac{L^1}{L^1 T^{-2}}} = \sqrt{L^1 \times L^{-1} T^{+2}}$$

$$M^0 L^0 T^1 = \sqrt{L^{1-1} T^2} = \sqrt{L^0 T^2} = \sqrt{T^2}$$

$$= T$$

$$M^0 L^0 T^1 = M^0 L^0 T^1$$

$\Rightarrow$ is correct.

Q Check $V = \sqrt{\frac{2GM}{R}}$.

$$M^0 L^1 T^{-1} = \sqrt{\frac{M^{-1} L^3 T^{-2} \times M^1}{L^1}} = \sqrt{M^0 L^2 T^{-2}}$$

$$= \sqrt{(L^1 T^{-1})^2} = L^1 T^{-1} = M^0 L^1 T^{-1}$$

$\Rightarrow$ is correct.

Q Find the dimensions of (i) a & b (ii) $a \times b$.
 (iii) $\frac{a}{b}$ in the equation $F = a\sqrt{x} + bt^2$,
 where F is force, x is distance & t is time.

so

$$a\sqrt{x} = F$$

$$a = \frac{F}{x^{1/2}} = \frac{M^1 L^1 T^{-2}}{(L^1)^{1/2}} = M^1 L^1 T^{-2} \times L^{-1/2} \\ = M^1 L^{1/2} T^{-2}$$

$$[a] = [M^1 L^{1/2} T^{-2}]$$

Now

$$bt^2 = [F]$$

$$b = \frac{F}{t^2} = \frac{M^1 L^1 T^{-2}}{T^2} = M^1 L^1 T^{-2} \times T^{-2}$$

$$[b] = [M^1 L^1 T^{-4}]$$

$$(ii) \quad a \times b = M^1 L^{1/2} T^{-2} \times M^1 L^1 T^{-4}$$

$$[a \times b] = [M^2 L^{3/2} T^{-6}]$$

$$(iii) \quad \frac{a}{b} = \frac{M^1 L^{1/2} T^{-2}}{M^1 L^1 T^{-4}} = M^1 L^{1/2} T^{-2} \times M^{-1} L^{-1} T^4$$

$$\left[\frac{a}{b}\right] = [M^0 L^{-1/2} T^2]$$



Q The Vander Wall's equation for a gas is

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT.$$

where P = Pressure, V = Volume, R = gas constant, T = temp

Find the dimensions of (i) a & b (ii) $a \times b$ (iii) $\frac{a}{b}$

Sol $\frac{a}{V^2} = P.$ [same units are added.]

$$a = P \times V^2 = M^1 L^{-1} T^{-2} \times [L^3]^2$$
$$= M^1 L^{-1+6} T^{-2}.$$

$$[a] = [M^1 L^5 T^{-2}]$$

Also

$$[b] = [V]$$
$$= L^3$$

$$[b] = [M^0 L^3 T^0]$$

$$(ii) \quad a \times b = M^1 L^5 T^{-2} \times L^3 = M^1 L^8 T^{-2}.$$

$$(iii) \quad \frac{a}{b} = \frac{M^1 L^5 T^{-2}}{L^3} = M^1 L^{5-3} T^{-2} = M^1 L^2 T^{-2}.$$



EXAMPLE 43. Check the dimensional consistency of the following equations :

(i) de-Broglie wavelength, $\lambda = \frac{h}{mv}$

(ii) Escape velocity, $v = \sqrt{\frac{2GM}{R}}$ [Himachal 05]

Solution. (i) Given $\lambda = \frac{h}{mv}$

As wavelength is a distance,

$$[\lambda] = L$$

$$\text{Also } \left[\frac{h}{mv} \right] = \frac{\text{Planck's constant}}{\text{Mass} \times \text{velocity}} = \frac{ML^2 T^{-1}}{M \times LT^{-1}} = L$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the given equation is dimensionally consistent.

(ii) Given $v = \sqrt{\frac{2GM}{R}}$

$$[v] = LT^{-1}$$

$$\left[\frac{2GM}{R} \right]^{\frac{1}{2}} = \left[\frac{M^{-1} L^3 T^{-2} \cdot M}{L} \right]^{\frac{1}{2}} = [L^2 T^{-2}]^{\frac{1}{2}} = LT^{-1}$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence the equation is dimensionally correct.

EXAMPLE 44. Check by the method of dimensions whether the following equations are correct :

(i) $E = mc^2$ (ii) $T = 2\pi \sqrt{\frac{l}{g}}$ [Delhi 98]

(iii) $v = \sqrt{\frac{P}{\rho}}$, where v = velocity of sound,

P = pressure and ρ = density of medium.

(iv) $v = \frac{1}{2l} \sqrt{\frac{T}{m}}$, where v = frequency of vibration,

l = length of the string, T = tension in the string and m = mass per unit length.

Solution.

(i) $[L.H.S.] = [E] = \text{Energy} = ML^2 T^{-2}$

$$[R.H.S.] = [mc^2] = M[LT^{-1}]^2 = ML^2 T^{-2}$$

Hence the relation is correct.

(ii) $[RHS] = \sqrt{\frac{[l]}{[g]}} = \left[\frac{L}{LT^{-2}} \right]^{\frac{1}{2}} = T$

$$[LHS] = [T] = T.$$

Hence the relation is correct.

(iii) $[RHS] = \sqrt{\frac{[P]}{[\rho]}} = \left[\frac{ML^{-1} T^{-2}}{ML^{-3}} \right]^{\frac{1}{2}} = LT^{-1}$

$$[LHS] = [v] = LT^{-1}.$$

Hence the relation is correct.

(iv) $RHS = \frac{1}{[l]} \sqrt{\frac{[T]}{[m]}} = \frac{1}{L} \left[\frac{MLT^{-2}}{ML^{-1}} \right]^{\frac{1}{2}}$

$$= \frac{1}{L} \cdot LT^{-1} = T^{-1}$$

$$[LHS] = [v] = T^{-1}.$$

Hence the relation is correct.

EXAMPLE 45. By the method of dimensions, test the accuracy of the equation :

$$\delta = \frac{mg l^3}{4b d^3 Y}$$

where δ is the depression produced in the middle of a bar of length l , breadth b and depth d , when it is loaded in the middle with mass m . Y is the Young's modulus of the material of the bar.

Solution. $[LHS] = [\delta] = \text{depression} = L$

$$[RHS] = \left[\frac{mg l^3}{4b d^3 Y} \right]$$

$$= \frac{\text{Mass} \times \text{acceleration} \times \text{length}^3}{4 \times \text{breadth} \times \text{depth}^3 \times \text{Young's modulus}}$$

$$= \frac{M \cdot LT^{-2} \cdot L^3}{L \cdot L^3 \cdot ML^{-1} T^{-2}} = L$$

$\therefore [LHS] = [RHS]$. Hence the relation is correct.

EXAMPLE 46. Find the dimensions of a/b in the equation :

$F = a\sqrt{x} + bt^2$, where F is force, x is distance and t is time.

Solution. $[a\sqrt{x}] = [F]$

$$\therefore [a] = \frac{[F]}{[\sqrt{x}]} = \frac{MLT^{-2}}{L^{1/2}} = ML^{1/2} T^{-2}$$

$$[bt^2] = [F]$$

$$\therefore [b] = \frac{[F]}{[t^2]} = \frac{MLT^{-2}}{T^2} = MLT^{-4}$$

$$[a/b] = \frac{ML^{1/2} T^{-2}}{MLT^{-4}} = L^{-1/2} T^2.$$



EXAMPLE 47. Find the dimensions of $a \times b$ in the relation :

$P = \frac{b-x^2}{at}$; where P is power, x is distance and t is time.

Solution. $[b] = [x^2] = L^2 \therefore [P] = \frac{L^2}{[at]}$

or $[a] = \frac{L^2}{[P][t]} = \frac{L^2}{ML^2 T^{-3} \cdot T} = M^{-1} T^2$

Hence $[a \times b] = M^{-1} L^2 T^2$.

EXAMPLE 48. The Vander Wall's equation for a gas is

$$\left(P + \frac{a}{V^2} \right) (V - b) = RT$$

Determine the dimensions of a and b . Hence write the SI units of a and b . [Himachal 06C; Central Schools 14]

Solution. Since dimensionally similar quantities can be added or subtracted, therefore,

$$[P] = \left[\frac{a}{V^2} \right]$$

or $[a] = [PV^2] = [ML^{-1} T^{-2}][L^3]^2 = ML^5 T^{-2}$

Also $[b] = [V] = L^3$

The SI unit of a is $kg\ m^5 s^{-2}$ and that of b is m^3 .

EXAMPLE 49. When white light travels through glass, the refractive index of glass (μ = velocity of light in air/velocity of light in glass) is found to vary with wavelength as $\mu = A + \frac{B}{\lambda^2}$. Using the principle of homogeneity of dimensions, find the SI units in which the constants A and B must be expressed.

Solution. Here $\mu = \frac{\text{Velocity of light in air}}{\text{Velocity of light in glass}}$

= a dimensionless number

$\therefore [A] = [\mu] = \text{a dimensionless number}$

As $\left[\frac{B}{\lambda^2} \right] = [\mu]$

$\therefore [B] = [\mu][\lambda^2] = 1 \cdot L^2 = L^2$

Hence A , being dimensionless, has no units and SI unit of B is m^2 .

EXAMPLE 50. In the equation : $y = a \sin(\omega t - kx)$, t and x stand for time and distance respectively. Obtain the dimensional formula for ω and k .

Solution. An angle is a dimensionless quantity,

$\therefore [\omega t] = 1 \quad \text{or} \quad [\omega] = \frac{1}{[t]} = \frac{1}{T} = T^{-1}$

$[kx] = 1 \quad \text{or} \quad [k] = \frac{1}{[x]} = \frac{1}{L} = L^{-1}$

EXAMPLE 51. Rule out or accept the following formulae for kinetic energy on the basis of dimensional arguments :

(i) $\frac{3}{16}mv^2$ (ii) $\frac{1}{2}mv^2 + ma$ [Central Schools 08]

Solution. K.E. = $\frac{1}{2}mv^2$

Dimensions of K.E. = $[ML^2T^{-2}]$

(i) Dimensions of $\frac{3}{16}mv^2 = [ML^2T^{-2}]$

This formula is dimensionally acceptable for K.E.

(ii) $\left[\frac{1}{2}mv^2 \right] + [ma] = [ML^2T^{-2}] + [MLT^{-2}]$

This formula cannot be accepted for K.E. as it is dimensionally inhomogeneous.

PROBLEMS FOR PRACTICE

- Test the dimensional consistency of the following equations : (i) $v = u + at$ (ii) $s = ut + \frac{1}{2}at^2$ (iii) $v^2 - u^2 = 2as$ [Himachal 07C]
(Ans. All relations are dimensionally correct)
- The viscous force ' F ' acting on a small sphere of radius ' r ' moving with velocity v through a liquid is given by $F = 6\pi\eta rv$. Calculate the dimensions of η , the coefficient of viscosity. (Ans. $ML^{-1}T^{-1}$)
- The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d . (Ans. $L, LT^{-1}, LT^{-2}, LT^{-3}$)
- The critical velocity of the flow of a liquid through a pipe of radius r is given by $v_c = \frac{K\eta}{r\rho}$ where ρ is the density and η is the coefficient of viscosity of the liquid. Check if this relation is dimensionally correct. (Ans. Correct)
- The rate of flow (V) of a liquid flowing through a pipe of radius r and a pressure gradient (P/l) is given by Poiseuille's equation : $V = \frac{\pi}{8} \frac{P r^4}{\eta l}$
Check the dimensional consistency of this equation. (Ans. Correct)
- Test if the following equation is dimensionally correct :
$$h = \frac{2S \cos \theta}{r\rho g}$$
 where h = height, S = surface tension, ρ = density, r = radius, and g = acceleration due to gravity. (Ans. Correct)
- Find the dimensions of the quantity v in the equation, $v = \frac{\pi p(a^2 - x^2)}{2\eta l}$, where a is the radius and l is the length of the tube in which the fluid of coefficient of viscosity η is flowing, x is the distance from the axis of the tube and p is the pressure difference. (Ans. LT^{-1})
- Find the dimensions of the quantity q from the expression. $T = 2\pi \sqrt{\frac{ml^3}{3Yq}}$, where T is the time period of a bar of length l , mass m and Young's modulus Y . (Ans. L^4)
- An artificial satellite of mass m is revolving in a circular orbit around a planet of M and radius R . If the radius of the orbit of the satellite be r . Justify by the method of dimensions that the time period of the satellite is given by : $T = \frac{2\pi}{R} \sqrt{\frac{r^3}{g}}$

10. Find the dimensions of $(a \times b)$ in the equation :
 $E = \frac{b - x^2}{at}$; where E is energy, x is distance and t is time.
 (Ans. $M^{-1} L^2 T$)

11. Find the dimensions of (a/b) in the equation :
 $P = \frac{a - t^2}{bx}$

where P is pressure, x is distance and t is time.

(Ans. MT^{-2})

12. Time period of an oscillating drop of radius r , density ρ and surface tension S is : $t = K \sqrt{\frac{\rho r^3}{S}}$

Check the correctness of the relation. [Himachal 04]

(Ans. correct)

13. Out of formulae (i) $y = a \sin 2\pi t / T$ and (ii) $y = a \sin vt$ for the displacement y of particle undergoing a certain periodic motion, rule out the wrong formula on dimensional grounds. (where a = maximum displacement of the particle, v = speed of the particle, T = time period of motion). [Delhi 09]

X HINTS

1. (i) $[v] = LT^{-1}$, $[u] = LT^{-1}$,
 $[at] = LT^{-2} \cdot T = LT^{-1}$

- (ii) $[s] = L$, $[ut] = LT^{-1} \cdot T = L$,

$$\left[\frac{1}{2} at^2 \right] = LT^{-2} \cdot T^2 = L$$

- (iii) $[v^2] = [LT^{-1}]^2 = L^2 T^{-2}$,
 $[u^2] = [LT^{-1}]^2 = L^2 T^{-2}$

$$[2as] = LT^{-2} \cdot L = L^2 T^{-2}$$

4. $[v_c] = \text{critical velocity} = LT^{-1}$

$$\left[\frac{K\eta}{r\rho} \right] = \frac{1 \cdot ML^{-1} T^{-1}}{L \cdot ML^{-3}} = LT^{-1}$$

Hence the relation is correct.

5. $[V] = \text{Rate of flow of liquid}$

$$= \text{Volume} / \text{Time} = L^3 T^{-1}$$

$$\left[\frac{\pi Pr^4}{8 \eta l} \right] = \frac{ML^{-1} T^{-2} \cdot L^4}{ML^{-1} T^{-1} \cdot L} = L^3 T^{-1}$$

Hence the relation is correct.

6. $[h] = L$

$$\left[\frac{2S \cos \theta}{r\rho g} \right] = \frac{MT^{-2} \cdot 1}{L \cdot ML^{-3} \cdot LT^{-2}} = L$$

Hence the relation is correct.

7. $[v] = \frac{[ML^{-1} T^{-2}] \cdot [L^2 - L^2]}{[ML^{-1} T^{-1}] \cdot [L]} = \frac{ML^{-1} T^{-2} \cdot L^2}{MT^{-1}} = LT^{-1}$

$$8. [q] = \left[\frac{4\pi^2 ml^3}{3YT^2} \right] = \frac{ML^3}{ML^{-1} T^{-2} \cdot T^2} = L^4$$

9. $[T] = T$

$$\left[\frac{2\pi}{R} \sqrt{\frac{r^3}{g}} \right] = \frac{1}{L} \left[\frac{L^3}{LT^{-2}} \right]^{1/2} = T$$

Hence the relation is correct.

11. $[a] = [t^2] = T^2 \therefore [P] = \frac{T^2}{[bx]}$

$$\text{or } [b] = \frac{T^2}{[P][x]} = \frac{T^2}{ML^{-1} T^{-2} \cdot L} = M^{-1} T^4$$

$$[a/b] = \frac{T^2}{M^{-1} T^4} = MT^{-2}$$

12. $[RHS] = \left[K \sqrt{\frac{\rho r^3}{S}} \right] = \left[\frac{[ML^{-3}][L^3]}{[MT^{-2}]} \right]^{1/2} = [T] = [LHS]$

13. Refer to the answer of NCERT Exercise 2.14.

2.26 DEDUCING RELATION AMONG THE PHYSICAL QUANTITIES

48. How can the method of dimensions be used to deduce a relation among the physical quantities? Explain it with the help of a suitable example.

To derive the relationship among physical quantities. By making use of the homogeneity of dimensions, we can derive an expression for a physical quantity if we know the various factors on which it depends.

Example. Let us derive an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r .

$$\text{Let } F \propto m^a v^b r^c \text{ or } F = K m^a v^b r^c \quad \dots(1)$$

where K is a dimensionless constant. Writing the dimensions of various quantities in equation (1), we get

$$[MLT^{-2}] = 1 [M]^a [LT^{-1}]^b [L]^c$$

$$\text{or } M^1 L^1 T^{-2} = M^a L^{b+c} T^{-b}$$

Comparing the dimensions of similar quantities on both sides, we get

$$a = 1$$

$$b + c = 1 \quad \text{or} \quad c = 1 - b = 1 - 2 = -1$$

$$-2 = -b \quad \text{or} \quad b = 2$$

From equation (1), we get

$$F = K m v^2 r^{-1} = K \frac{mv^2}{r}$$

This is the required expression for the centripetal force.

Derivation of the formulae:

- Q If centripetal force F depends on mass^m of particle, velocity v of particle & radius of circle. $2\pi r$.
Find the expression for centripetal force F using dimensions.

SA

$$F \propto m^a v^b r^c.$$

$$F = K m^a v^b r^c \text{ --- (1)}$$

where K = dimensionless constant.

$$M^1 L^1 T^{-2} = [M]^a [L T^{-1}]^b [L]^c.$$

$$= M^a L^b T^{-b} L^c.$$

$$M^1 L^1 T^{-2} = M^a L^{b+c} T^{-b}.$$

Comparing

$a=1$	$b+c=1$	$+b=+2.$
	$2+c=1$	$b=2$
	$c=1-2$	
	$c=-1$	

Using value of a, b & c in eq (1)

$$F = K m^1 v^2 r^{-1} = K \frac{mv^2}{r}$$

By other methods, $K=1$

$$\therefore F = \frac{mv^2}{r} = \text{centripetal force.}$$

P-01.
28/4
11th

Q. Suppose the period of oscillation of simple pendulum depends on (i) mass m of the bob (ii) length l of the pendulum and (iii) acceleration due to gravity g at the place. Derive the expression for its time period using method of dimensions.

sa $T \propto m^a l^b g^c$

$$T = K m^a l^b g^c \quad \text{--- (1)}$$

, K = Dimensionless constant

$$M^0 L^0 T^1 = [M]^a [L]^b [L T^{-2}]^c$$

$$M^0 L^0 T^1 = M^a L^b L^c T^{-2c}$$

$$M^0 L^0 T^1 = M^a L^{b+c} T^{-2c}$$

Comparing

$$\boxed{a=0} \quad \left| \quad b+c=0 \quad \left| \quad -2c=1 \right. \right. \\ \left| \quad b-\frac{1}{2}=0 \quad \left| \quad c=-\frac{1}{2} \right. \right. \\ \boxed{b=\frac{1}{2}}$$

Using the values of a, b & c in eq (1)

$$T = K m^0 l^{1/2} g^{-1/2} = K \frac{l^{1/2}}{g^{1/2}} = K \left(\frac{l}{g} \right)^{1/2}$$

$$T = K \sqrt{\frac{l}{g}}$$

By other methods
 $K = 2\pi$

$$\therefore \boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

P-2
28/4
11th

Q The wavelength λ associated with a moving electron depends on its mass m , its velocity v & Planck's constant h . Prove dimensionally that $\lambda = \frac{h}{mv}$. (Take $K=1$)

Sol

$$\lambda \propto m^a v^b h^c$$

$$\lambda = K m^a v^b h^c \quad \text{--- (1)}$$

$$M^0 L^1 T^0 = [M]^a [L T^{-1}]^b [M L^2 T^{-1}]^c$$

$$= M^a L^b T^{-b} M^c L^{2c} T^{-c}$$

$$M^0 L^1 T^0 = M^{a+c} L^{b+2c} T^{-b-c}$$

comp.

$$a+c=0$$

$$\left| \begin{array}{l} b+2c=1 \\ -b-c=0 \end{array} \right| \quad \text{--- (ii) \quad (iii)}$$

Since $c=1$

$$\therefore a+1=0$$

$$\boxed{a=-1}$$

Add $\times$ (ii) & (iii)
 $c=1$

$$b+2(1)=1$$

$$b=1-2$$

$$\boxed{b=-1}$$

$$\begin{array}{r} b+2c=1 \\ -b-c=0 \\ \hline \end{array}$$

$$\boxed{c=1}$$

From (1)

$$\therefore \lambda = K m^{-1} v^{-1} h^1 = K \frac{h}{mv}$$

~~As~~ $K=1$

$$\boxed{\lambda = \frac{h}{mv}}$$

$$h = M^1 L^2 T^{-1}$$

P-01
29/4
11m

Q A small spherical ball of radius r falls with velocity v through a liquid having coeff of viscosity η . Find the viscous drag F on the ball assuming it depends on η, r & v .
(Stoke's law) $K=6\pi$.

sd $F \propto r^a v^b \eta^c$

$F = K r^a v^b \eta^c$ — ① viscous drag

$M^1 L^1 T^{-2} = [L]^a [L^1 T^{-1}]^b [M^1 L^{-1} T^{-1}]^c$

$= L^a L^b T^{-b} M^c L^{-c} T^{-c}$

$M^1 L^1 T^{-2} = M^c L^{a+b-c} T^{-b-c}$

$c=1$

$a+b-c=1$ $-b-c=-2$
 $\therefore a+1-1=1$ $-b-1=-2$

$a+0=1$

$a=1$

$-b=-2+1$

$-b=-1$

$b=1$

$\therefore F \propto$ ①

$F = K r^1 v^1 \eta^1$

$F = 6\pi \eta r v$

→ Stokes law

$\eta = M^1 L^{-1} T^{-1}$

P-2
29/4
11A

Q Reynold's Number N_R is a function of density of the liquid ρ , its average speed V , coeff of viscosity η & diameter D of the pipe. Given that N_R is directly prop to D . Derive the expression for Reynold's No N_R using dimensional analysis.

Q $N_R \propto \rho^a V^b \eta^c D^d$

But N_R is directly prop to D i.e. $d=1$.

$\therefore N_R = K \rho^a V^b \eta^c D^1$ — (1)

$$M^0 L^0 T^0 = [M^1 L^{-3}]^a [L^1 T^{-1}]^b [M^1 L^{-1} T^{-1}]^c L^1$$

$$= M^a L^{-3a} L^b T^{-b} M^c L^{-c} T^{-c} L^1$$

$$M^0 L^0 T^0 = M^{a+c} L^{-3a+b-c+1} T^{-b-c}$$

comp.

$$\begin{array}{l|l|l} a+c=0 & -3a+b-c+1=0 & -b-c=0 \\ a=-c & \rightarrow -3(-c)-c-c+1=0 & \leftarrow \begin{array}{l} -b=c \\ b=-c \end{array} \end{array}$$

$$a = -(-1) \quad 3c - 2c + 1 = 0$$

$$\boxed{a=1}$$

$$c+1=0$$

$$\boxed{c=-1}$$

$$\therefore b = -(-1)$$

$$\boxed{b=1}$$

$\therefore$ From (1)

$$N_R = K \rho^1 V^1 \eta^{-1} D^1$$

$$\Rightarrow \boxed{N_R = K \frac{\rho V D}{\eta}}$$

P-3
29/14
nm

Examples based on Deriving Relationship between Physical Quantities

CONCEPT USED

By using the principle of homogeneity of dimensions, the form of expression for a given physical quantity can be obtained if we know the factors upon which that physical quantity depends.

EXAMPLE 52. Consider a simple pendulum, having a bob attached to a string, that oscillates under the action of the force of gravity. Suppose that the period of oscillation of the simple pendulum depends on (i) mass m of the bob (ii) length l of the pendulum and (iii) acceleration due to gravity g at the place. Derive the expression for its time period using method of dimensions.

[NCERT]

Solution. Let us assume that $T \propto m^a l^b g^c$

$$\text{or } T = Km^a l^b g^c \quad \dots(i)$$

where K is a dimensionless constant.

The dimensions of various quantities are

$$[T] = T, [m] = M, [l] = L, [g] = LT^{-2}$$

Substituting these dimensions in eqn. (i), we get

$$T = [M]^a [L]^b [LT^{-2}]^c$$

$$\text{or } M^0 L^0 T^1 = M^a L^{b+c} T^{-2c}$$

Equating the exponents of M , L and T on both sides, we get

$$a = 0, \quad b + c = 0, \quad -2c = 1$$

$$\text{On solving, } a = 0, \quad b = \frac{1}{2}, \quad c = -\frac{1}{2}$$

$$\therefore T = Km^0 l^{1/2} g^{-1/2} = K \sqrt{\frac{l}{g}}$$

From experiments, $K = 2\pi$

$$\text{Therefore } T = 2\pi \sqrt{\frac{l}{g}}$$

EXAMPLE 53. The velocity ' v ' of water waves depends on the wavelength ' λ ', density of water ' ρ ' and the acceleration due to gravity ' g '. Deduce by the method of dimensions the relationship between these quantities. [Central Schools 08]

Solution. Let $v = K \lambda^a \rho^b g^c$...(i)

where $K = a$ dimensionless constant.

Dimensions of the various quantities are

$$[v] = LT^{-1}, [\lambda] = L, [\rho] = ML^{-3}, [g] = LT^{-2}$$

Substituting these dimensions in equation (i), we get

$$[LT^{-1}] = [L]^a [ML^{-3}]^b [LT^{-2}]^c$$

$$M^0 L^1 T^{-1} = M^b L^{a-3b+c} T^{-2c}$$

Equating the powers of M , L and T on both sides,

$$b = 0, \quad a - 3b + c = 1, \quad -2c = -1$$

$$\text{On solving, } a = \frac{1}{2}, \quad b = 0, \quad c = \frac{1}{2}$$

$$\therefore v = K \lambda^{\frac{1}{2}} \rho^0 g^{\frac{1}{2}} = K \sqrt{\lambda g}$$

✱

EXAMPLE 54. Assuming that the mass M of the largest stone that can be moved by a flowing river depends upon ' v ' the velocity, ' ρ ' the density of water and on ' g ', the acceleration due to gravity. Show that M varies with the sixth power of the velocity of flow.

Solution. Let $M = Kv^a \rho^b g^c$...(i)

where $K = a$ dimensionless constant.

Dimensions of the various quantities are :

$$[M] = M, [v] = LT^{-1}, [\rho] = ML^{-3}, [g] = LT^{-2}$$

Substituting these dimensions in equation (i), we get

$$[M] = [LT^{-1}]^a [ML^{-3}]^b [LT^{-2}]^c$$

$$M^1 L^0 T^0 = M^b L^{a-3b+c} T^{-a-2c}$$

Equating the powers of M , L and T , we get

$$b = 1, \quad a - 3b + c = 0, \quad -a - 2c = 0$$

$$\text{On solving, } a = 6, \quad b = 1, \quad c = -3$$

$$\therefore M = Kv^6 \rho^1 g^{-3}$$

$$\text{Hence } M \propto v^6$$

EXAMPLE 55. The velocity of sound waves ' v ' through a medium may be assumed to depend on :

(i) the density of the medium ' d ' and

(ii) the modulus of elasticity ' E '.

Deduce by the method of dimensions the formula for the velocity of sound. Take dimensional constant $K = 1$.

Solution. Let the velocity of sound waves be given by

$$v = K d^a E^b \quad \dots(i)$$

where $K = a$ dimensionless constant.

Dimensions of the various quantities are

$$[v] = LT^{-1}, [d] = ML^{-3}$$

$$[E] = \frac{\text{stress}}{\text{strain}} = \frac{\text{force}}{\text{area} \times \text{strain}} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

Substituting these dimensions in equation (i), we get

$$[LT^{-1}] = [ML^{-3}]^a [ML^{-1}T^{-2}]^b$$

$$\text{or } M^0 L^1 T^{-1} = M^{a+b} L^{-3a-b} T^{-2b}$$

Equating the dimensions of M , L and T , we get

$$a + b = 0, \quad -3a - b = 1, \quad -2b = -1$$

$$\text{On solving, } a = -\frac{1}{2}, \quad b = \frac{1}{2}$$

$$\therefore v = K d^{-1/2} E^{1/2} \quad \text{or} \quad v = \sqrt{\frac{E}{d}} \quad [\because K = 1]$$

EXAMPLE 56. The frequency ' ν ' of vibration of a stretched string depends upon :

- its length l ,
- its mass per unit length ' m ' and
- the tension T in the string.

Obtain dimensionally an expression for frequency ν .

[Delhi 2002]

Solution. Let the frequency of vibration of the string be given by

$$\nu = K l^a m^b T^c \quad \dots(i)$$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$$[\nu] = T^{-1}, [l] = L, [T] = \text{Force} = MLT^{-2}$$

$$[m] = \frac{\text{mass}}{\text{length}} = \frac{M}{L} = ML^{-1},$$

Substituting these dimensions in equation (i), we get

$$[T^{-1}] = [L]^a [ML^{-1}]^b [MLT^{-2}]^c$$

$$\text{or } M^0 L^0 T^{-1} = M^{b+c} L^{a-b+c} T^{-2c}$$

Equating the dimensions of M , L and T , we get

$$b+c=0, \quad a-b+c=0, \quad -2c=-1$$

$$\text{On solving, } a=-1, \quad b=-\frac{1}{2}, \quad c=\frac{1}{2}.$$

$$\therefore \nu = K l^{-1} m^{-1/2} T^{1/2}$$

$$\text{or } \nu = \frac{K}{l} \sqrt{\frac{T}{m}}.$$

EXAMPLE 57. A planet moves around the sun in nearly circular orbit. Its period of revolution ' T ' depends upon :

- radius ' r ' of orbit
- mass ' M ' of the sun and
- the gravitational constant G .

Show dimensionally that $T^2 \propto r^3$.

Solution. Let $T = K r^a M^b G^c$

where K = a dimensionless constant.

Dimensions of the various quantities are :

$$[T] = T, [r] = L, [M] = M$$

$$[G] = \frac{Fr^2}{m_1 m_2} = \frac{MLT^{-2} \cdot L^2}{MM} = M^{-1} L^3 T^{-2}.$$

Substituting these dimensions in equation (i), we get

$$[T] = [L]^a [M]^b [M^{-1} L^3 T^{-2}]^c$$

$$M^0 L^0 T^1 = M^{b-c} L^{a+3c} T^{-2c}$$

Equating the dimensions of M , L and T , we get

$$b-c=0, \quad a+3c=0, \quad -2c=1$$

On solving

$$a=\frac{3}{2}, \quad b=-\frac{1}{2}, \quad c=-\frac{1}{2}$$

$$\therefore T = K r^{3/2} M^{-1/2} G^{-1/2}$$

$$\text{or } T^2 = \frac{K^2 r^3}{MG}$$

$$\therefore T^2 \propto r^3.$$

EXAMPLE 58. Reynold number N_R (a dimensionless quantity) determines the condition of laminar flow of a viscous liquid through a pipe. N_R is a function of the density of the liquid ' ρ ', its average speed ' v ' and coefficient of viscosity ' η '. Given that N_R is also directly proportional to ' D ' (the diameter of the pipe), show from dimensional considerations that $N_R \propto \frac{\rho v D}{\eta}$.

The unit of η in SI system is $kg m^{-1} s^{-1}$.

Solution. Let $N_R = K \rho^a v^b \eta^c D$...(i)

where K = a dimensionless constant.

Dimensions of various quantities are

$$[N_R] = 1 = M^0 L^0 T^0$$

$$[\rho] = ML^{-3}, [v] = LT^{-1},$$

$$[\eta] = ML^{-1} T^{-1}, [D] = L$$

Substituting these dimensions in equation (i), we get

$$[M^0 L^0 T^0] = [ML^{-3}]^a [LT^{-1}]^b [ML^{-1} T^{-1}]^c [L]$$

$$M^0 L^0 T^0 = M^{a+c} L^{-3a+b-c+1} T^{-b-c}$$

Equating the powers of M , L and T , we get

$$a+c=0, \quad -3a+b-c+1=0, \quad -b-c=0$$

On solving, $a=1, \quad b=1, \quad c=-1$.

$$\therefore N_R = K \rho^1 v^1 \eta^{-1} D = K \frac{\rho v D}{\eta}$$

$$\text{or } N_R \propto \frac{\rho v D}{\eta}.$$

EXAMPLE 59. Derive by the method of dimensions, an expression for the volume of a liquid flowing out per second through a narrow pipe. Assume that the rate of flow of liquid depends on

- the coefficient of viscosity ' η ' of the liquid
- the radius ' r ' of the pipe and
- the pressure gradient $\left(\frac{p}{l}\right)$ along the pipe.

$$\text{Take } K = \frac{\pi}{8}.$$

Solution. Let volume flowing out per second through the pipe be given by

$$V = K \eta^a r^b \left(\frac{p}{l}\right)^c \quad \dots(i)$$

where K = a dimensionless constant.

Dimensions of the various quantities are

$$[V] = \frac{\text{volume}}{\text{time}} = \frac{L^3}{T} = L^3 T^{-1}$$

$$[\eta] = ML^{-1} T^{-1}, [r] = L$$

$$\left[\frac{p}{l} \right] = \frac{\text{pressure}}{\text{length}} = \frac{\text{force}}{\text{area} \times \text{length}} = \frac{MLT^{-2}}{L^2 \cdot L} = ML^{-2} T^{-2}$$

Substituting these dimensions in equation (i), we get

$$[L^3 T^{-1}] = [ML^{-1} T^{-1}]^a [L]^b [ML^{-2} T^{-2}]^c$$

$$\text{or } M^0 L^3 T^{-1} = M^{a+c} L^{-a+b-2c} T^{-a-2c}$$

Equating the powers of M, L and T, we get

$$a+c=0, -a+b-2c=3, -a-2c=-1$$

On solving,

$$a=-1, b=4, c=1$$

$$\therefore V = K \eta^{-1} r^4 \left(\frac{p}{l} \right)^1$$

$$\text{or } V = \frac{\pi r^4 p}{8\eta l} \quad [\text{Poiseuille's equation}]$$

EXAMPLE 60. The period of vibration of a tuning fork depends on the length l of its prong, density d and Young's modulus Y of its material. Deduce an expression for the period of vibration on the basis of dimensions.

$$\text{Solution. Let } T = K l^a d^b Y^c \quad \dots(i)$$

where K = a dimensionless constant.

Dimensions of various quantities are

$$[T] = T, [l] = L, [d] = ML^{-3}, [Y] = ML^{-1} T^{-2}$$

Substituting these dimensions in equation (i), we get

$$T = [L]^a [ML^{-3}]^b [ML^{-1} T^{-2}]^c$$

$$\text{or } M^0 L^0 T^1 = M^{b+c} L^{a-3b-c} T^{-2c}$$

Equating the powers of M, L and T, we get

$$b+c=0, a-3b-c=0, -2c=1$$

On solving,

$$a=1, b=\frac{1}{2}, c=-\frac{1}{2}$$

$$\therefore T = K l^1 d^{1/2} Y^{-1/2} = K l \sqrt{\frac{d}{Y}}$$

EXAMPLE 61. The frequency ν of an oscillating drop may depend upon radius r of the drop, density ρ of the liquid and surface tension S of the liquid. Establish an expression for ν dimensionally.

[Delhi 13]

$$\text{Solution. Let } \nu = K r^a \rho^b S^c \quad \dots(i)$$

where K = a dimensionless constant.

Dimensions of various quantities are

$$[\nu] = T^{-1}, [r] = L, [\rho] = ML^{-3}, [S] = MT^{-2}$$

Substituting these dimensions in equation (i), we get

$$T^{-1} = [L]^a [ML^{-3}]^b [MT^{-2}]^c$$

$$\text{or } M^0 L^0 T^{-1} = M^{b+c} L^{a-3b} T^{-2c}$$

Equating the powers of M, L and T on both sides,

$$b+c=0, a-3b=0, -2c=-1$$

On solving,

$$a=-\frac{3}{2}, b=-\frac{1}{2}, c=\frac{1}{2}$$

$$\therefore \nu = K r^{-3/2} \rho^{-1/2} S^{1/2} = K \sqrt{\frac{S}{\rho r^3}}$$

EXAMPLE 62. The escape velocity v of a body depends upon (i) the acceleration due to gravity of the planet and (ii) the radius of the planet R . Establish dimensionally the relationship between v , g and R . [Himachal 06 ; Delhi 05]

$$\text{Solution. Let } v = K g^a R^b$$

where K = a dimensionless constant.

Putting the dimensions,

$$LT^{-1} = [LT^{-2}]^a [L]^b = L^{a+b} T^{-2a}$$

Equating the powers of L and T,

$$a+b=1, -2a=-1$$

$$\therefore a=\frac{1}{2}, b=\frac{1}{2}$$

$$\text{Hence } v = K g^{1/2} R^{1/2} = K \sqrt{gR}$$

EXAMPLE 63. A large fluid star oscillates in shape under the influence of its own gravitational field. Using dimensional analysis, find the expression for period of oscillation (T) in terms of radius of star (R), mean density of fluid (ρ) and universal gravitational constant G .

[Chandigarh 04]

$$\text{Solution. Let } T = K R^a \rho^b G^c$$

where K is a dimensionless constant.

Putting the dimensions,

$$T^1 = [L]^a [ML^{-3}]^b [M^{-1} L^3 T^{-2}]^c$$

$$\text{or } M^0 L^0 T^1 = M^{b-c} L^{-3b+3c+a} T^{-2c}$$

Equating the powers of M, L and T, we get

$$b-c=0, -3b+3c+a=0, -2c=1$$

On solving,

$$a=0, b=-\frac{1}{2}, c=-\frac{1}{2}$$

$$\therefore T = K R^0 \rho^{-1/2} G^{-1/2} = K \frac{1}{\sqrt{\rho G}}$$

PROBLEMS FOR PRACTICE

1. The wavelength λ associated with a moving electron depends on its mass m , its velocity v and Planck's constant h . Prove dimensionally that $\lambda \propto \frac{h}{mv}$.

[Himachal 04 ; Chandigarh 08]

2. Obtain an expression for the centripetal force F acting on a particle of mass m moving with velocity v in a circle of radius r . Take dimensionless constant $K = 1$.

[Himachal 2000]

$$\left(\text{Ans. } F = \frac{mv^2}{r} \right)$$

3. The orbital velocity v of a satellite may depend on its mass m , the distance r from the centre of the earth and acceleration due to gravity g . Obtain an expression for its orbital velocity. (Ans. $v = K\sqrt{rg}$)

4. A small spherical ball of radius r falls with velocity v through a liquid having coefficient of viscosity η . Find the viscous drag F on the ball assuming it depends on η , r and v . Take $K = 6\pi$.

$$(\text{Ans. } F = 6\pi\eta rv)$$

5. The velocity of a freely falling body is a function of the distance fallen through (h) and acceleration due to gravity g . Show by the method of dimensions that $v = K\sqrt{gh}$.

6. Using the method of dimensions, derive an expression for the energy of a body executing SHM ; assuming this energy depends upon its mass m , frequency ν and amplitude of vibration r .

[Himachal 06]

$$(\text{Ans. } E = Kmv^2 r^2)$$

7. A body of mass m hung at one end of the spring executes SHM. Prove that the relation $T = 2\pi m/k$ is incorrect, where k is the force constant of the spring. Also derive the correct relation. (Ans. $T = K\sqrt{m/k}$)

8. Assuming that the critical velocity v_c of a viscous liquid flowing through a capillary tube depends only upon the radius r of the tube, density ρ and the coefficient of viscosity η of the liquid, find the expression for critical velocity.

$$\left(\text{Ans. } v_c = \frac{K\eta}{r\rho} \right)$$

9. By the method of dimensions, obtain an expression for the surface tension S of a liquid rising in a capillary tube. Assume that the surface tension depends upon (i) mass m of the liquid (ii) pressure p of the liquid and (iii) radius r of the capillary tube. Take $K = 1/2$.

$$(\text{Ans. } S = pr/2)$$

10. The depth x to which a bullet penetrates a human body depends upon (i) coefficient of elasticity η and (ii) kinetic energy E_k . By the method of dimensions, show that : $x \propto \left[\frac{E_k}{\eta} \right]^{1/3}$.

11. A U-tube of uniform cross-section contains mercury upto a height h in either limb. The mercury in one limb is depressed a little and then released. Obtain an expression for the time period of oscillation assuming that T depends on h , ρ and g .

$$\left(\text{Ans. } T = K\sqrt{\frac{h}{g}} \right)$$

12. The critical angular velocity ω_c of a cylinder inside another cylinder containing a liquid at which its turbulence occurs depends on viscosity η , density ρ and the distance d between the walls of the cylinder. Find an expression for ω_c .

$$\left(\text{Ans. } \omega_c = \frac{K\eta}{\rho d^2} \right)$$

13. A body of mass m is moving in a circle of radius r with angular velocity ω . Find expression for centripetal force acting on it by the method of dimensions. [Himachal 03, 09C] (Ans. $F = Kmr\omega^2$)

14. Consider a simple pendulum. The period of oscillation of the simple pendulum depends on its length ' l ' and acceleration due to gravity ' g '. Derive the expression for its period of oscillation by the method of dimensions.

[Himachal 06, 06C, 07]

HINTS

1. Let $\lambda = Km^a v^b h^c$, then

$$M^0 L^1 T^0 = [M]^a [L T^{-1}]^b [M L^2 T^{-1}]^c$$

$$= M^{a+c} L^{b+2c} T^{-b-c}$$

$$\therefore a+c=0, \quad b+2c=1, \quad -b-c=0$$

$$\text{On solving, } a=-1, \quad b=-1, \quad c=1$$

$$\therefore \lambda = k \frac{h}{mv} \quad \text{or} \quad \lambda \propto \frac{h}{mv}$$

4. Let $F = K\eta^a r^b v^c$, then

$$M^1 L^1 T^{-2} = [M L^{-1} T^{-1}]^a [L]^b [L T^{-1}]^c$$

$$= M^a L^{-a+b+c} T^{-a-c}$$

$$\therefore a=1, \quad -a+b+c=1, \quad -a-c=-2$$

$$\text{On solving, } a=b=c=1$$

$$\text{Hence } F = K\eta rv = 6\pi\eta rv. \quad (\text{Stoke's law})$$

6. Let $E = Km^a \nu^b r^c$, then

$$M^1 L^2 T^{-2} = [M]^a [T^{-1}]^b [L]^c = M^a L^c T^{-b}$$

$$\therefore a=1, \quad b=2, \quad c=2 \quad \text{Hence } E = Kmv^2 r^2.$$

7. [LHS] = [T] = T

$$[\text{RHS}] = \left[\frac{2\pi m}{k} \right] = \frac{M}{MT^{-2}} = T^2$$

$$\therefore [k] = \frac{\text{Force}}{\text{Length}} = \frac{MLT^{-2}}{L} = MT^{-2}$$

$$[\text{LHS}] \neq [\text{RHS}]. \quad \text{Hence the relation is incorrect.}$$

To find the correct relation, let $T = K m^a k^b$, then

$$T^1 = [M]^a [MT^{-2}]^b = M^{a+b} T^{-2b}$$

$$\therefore a + b = 0, -2b = 1$$

$$\text{On solving, } a = \frac{1}{2}, b = -\frac{1}{2}$$

$$\text{Hence } T = K m^{1/2} k^{-1/2} = K \sqrt{\frac{m}{k}}$$

8. Let $v_c = K r^a \rho^b \eta^c$, then

$$LT^{-1} = [L]^a [ML^{-3}]^b [ML^{-1} T^{-1}]^c \\ = M^{b+c} L^{a-3b-c} T^{-c}$$

$$\therefore b + c = 0, a - 3b - c = 1, -c = -1$$

On solving,

$$a = -1, b = -1, c = 1$$

$$\therefore v_c = K r^{-1} \rho^{-1} \eta^1 = \frac{K \eta}{r \rho}$$

10. Let $x = K \eta^a [E_k]^b$, then

$$L^1 = [ML^{-1} T^{-2}]^a [ML^2 T^{-2}]^b \\ = M^{a+b} L^{-a+2b} T^{-2a-2b}$$

$$\therefore a + b = 0, -a + 2b = 1, -2a - 2b = 0$$

$$\text{On solving, } a = -\frac{1}{3}, b = \frac{1}{3}$$

$$\text{Hence } x = K \eta^{-1/3} E_k^{1/3} = K \left[\frac{E_k}{\eta} \right]^{1/3}$$

12. Let $\omega_c = K \eta^a \rho^b d^c$, then

$$T^{-1} = [ML^{-1} T^{-1}]^a [ML^{-3}]^b [L]^c \\ = M^{a+b} L^{-a-3b+c} T^{-a}$$

$$\therefore a + b = 0, -a - 3b + c = 0, -a = -1$$

On solving,

$$a = 1, b = -1, c = -2$$

$$\text{Hence } \omega_c = K \eta^1 \rho^{-1} d^{-2} = \frac{K \eta}{\rho d^2}$$

2.27 ▼ LIMITATIONS OF DIMENSIONAL ANALYSIS

49. Mention the limitations of the method of dimensional analysis.

Limitations of the method of dimensions :

- ✓ 1. The method does not give any information about the dimensionless constant K .
- ✓ 2. It fails when a physical quantity depends on more than three physical quantities.
3. It fails when a physical quantity (e.g., $s = ut + \frac{1}{2} at^2$) is the sum or difference of two or more quantities.

- ✓ 4. It fails to derive relationships which involve trigonometric, logarithmic or exponential functions.
5. Sometimes, it is difficult to identify the factors on which the physical quantity depends. The method becomes more complicated when dimensional constants like G, h , etc. are involved.

2.28 ▼ SIGNIFICANT FIGURES

50. What is meant by significant figures in a measured quantity?

Significant figures. The significant figures are normally those digits in a measured quantity which are known reliably or about which we have confidence in our measurement plus one additional digit that is uncertain. The larger the number of significant figures in a measurement, the higher is the accuracy of the measurement. Suppose the time period of a simple pendulum is 1.62s. This digits 1 and 6 are reliable and certain, while the digit 2 is uncertain. So the time period has three significant figures. Again, suppose the length of an object is measured as 273.6 cm. It has four significant figures. The digits 2, 7 and 3 are reliable while the digit 6 is uncertain.

51. State the rules for counting the number of significant figures in a measured quantity.

Rules for determining the number of significant figures :

- (i) All non-zero digits are significant. So 13.75 has four significant figures.
- (ii) All zeros between two non-zero digits are significant. Thus 100.05 km has five significant figures.
- (iii) All zeros to the right of a non-zero digit but to the left of an understood decimal point are not significant. For example, 86400 has three significant figures. But such zeros are significant if they come from a measurement. For example, 86400 s has five significant figures.
- (iv) All zeros to the right of a non-zero digit but to the left of a decimal point are significant. For example, 648700. has six significant figure.
- (v) All zeros to the right of a decimal point are significant. So 161 cm, 161.0 cm and 161.00 cm have three, four and five significant figures respectively.
- (vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant. So 0.161 cm and 0.0161 cm, both have three significant figures. Moreover, zero conventionally placed to the left of the decimal point is not significant.
- (vii) The number of significant figures does not depend on the system of units. So 16.4 cm, 0.164 m and 0.000164 km, all have three significant figures.